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A survey of machine learning and evolutionary computation for antenna modeling and optimization: Methods and challenges

Hanhua Zou [a], Sanyou Zeng [a,*], Changhe Li [b,*], Jingyu Ji [c]

[a] School of Mechanical Engineering and Electronic Information, China University of Geosciences, Wuhan, China
[b] School of Artificial Intelligence, Anhui University of Sciences & Technology, Hefei, China
[c] Department of Computing and Decision Sciences, Lingnan University, Hong Kong

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ABSTRACT

Antenna design is a kind of electromagnetic (EM) engineering problem and normally formulated as complex nonlinear optimization problem. Evolutionary computation (EC) was combined early to antenna design due to its powerful nonlinear optimization capability. Modern antenna design depends on EM simulation software to solve Maxwell equations, which is time-consuming and makes it nontrivial for application of EC in antenna design. Machine learning (ML) is widely used to accelerate antenna design by building surrogate model of EM simulation. However, existing surveys focus on one of these two artificial intelligence (AI) methods (EC and ML) in antenna applications, and have overlooked differences between two cases of surrogate model for antenna EM simulation (response modeling and specification modeling). This review paper aims to summarize the applications of both EC and ML in antenna design over the past decades and highlight advantages and disadvantages of two kinds of EM simulation surrogate models. The survey begins with a short overview of ML and EC basics. Then various applications are discussed in three parts, including antenna optimization with EC, ML-assisted antenna optimization with response modeling and ML-assisted antenna optimization with specification modeling. Finally, challenges and potential future directions for applying ML and EC in antenna design are discussed, as well as emerging trends. This survey provides a comprehensive introduction to ML and EC in antenna design and contributes to the investigation of AI-empowered antenna design.

1. Introduction

1.1. Antenna design basics

Antennas are critical signal transceiver devices for wireless communications, radar, and many other radio applications, which serve as converters that convert guided waves on transmission lines to radio waves in an unbounded medium (usually free space) when transmitting and vice versa when receiving (Balanis, 2016). The working principle of the antenna is illustrated in Fig. 1. When there is alternating current passing through the wire, the radiation of electromagnetic (EM) waves can occur, and the radiation capacity is related to the length and shape of the wire. If two wires are close together and the electric field is bound between the two wires, then the radiation is weak. If the two wires are open, the electric field is spread in the surrounding space, then the radiation is enhanced. It should be noted that when the length of the wire is much less than the wavelength, the radiation is very weak. When the length of the wire increase to be comparable to the wavelength, the current on the wire will greatly increase, forming a stronger radiation.

Antenna design generally analyzes and optimizes design parameters (geometry, excitation, array distribution, etc.) according to antenna scattering performance (voltage standing wave ratio (VSWR), reflection coefficient S_{11}, isolation, etc.) and radiation performance (gain, pattern, side-lobe level (SLL), etc.). Generally speaking, a good antenna design needs to minimize VSWR and S_{11}, which means less return loss and greater energy radiation efficiency. To minimize isolation achieves smaller mutual coupling effect between array elements. Also, maximizing the gain and minimizing SLL enable better directivity of radiation pattern. When it comes to beamforming or beam scanning, antenna pattern plays a key role to match the desired pattern that difference between patterns is minimized.

Notably, there are two forms to describe antenna performance, including response and specification. Response is often used to observe antenna performance characteristics, such as frequency characteristics of S_{11}, radiation characteristics of pattern, etc. Specifications are often used to formulate objectives and constraints of antenna design optimization problem. Difference between them is illustrated in Fig. 2. Take S_{11} as example, antenna design parameters are firstly analyzed by EM

* Corresponding authors.
E-mail addresses: hanhuazou@cug.edu.cn (H. Zou), sanyouzeng@qq.com (S. Zeng), changhe.lw@gmail.com (C. Li), jingyuji@ln.edu.hk (J. Ji).

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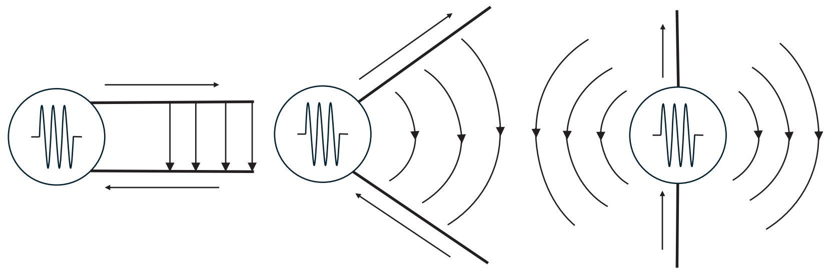


Fig. 1. Illustration of the working principle of the antenna.

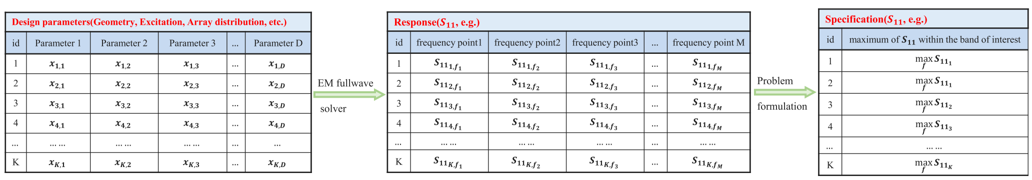


Fig. 2. Illustration of difference between response and specification (S_{11} is taken as example).

Table 1
Terminology correspondence in machine learning and evolutionary computation for antenna design.

	Input characteristics	Output characteristics	
Antenna design	Design parameters	Response	Specification
Machine learning	Features	Label	Label
Evolutionary computation	Decision variables	–	Objectives/Constraints

fullwave solver and S_{11} response within band of interest is obtained. Then specifications are calculated according to problem formulation based on response. For example, maximum of S_{11} within band of interest is normally formulated as specification to optimize S_{11} value over whole frequency band.

1.2. Traditional antenna design methods

To find best designs that fulfill the desired performance, it is a common practice in the early stage to fine-tune the geometric parameters of antenna structures for performance improvement based on “cut and try” and anechoic chamber measurement (Yaghjian, 1986). This is a time-consuming process of trial and error with high physical and labor cost, which is no longer suitable for contemporary antenna design since antenna structures become increasingly complex and make it difficult for engineers to make the right decisions based on experience.

Later, computational electromagnetics methods, such as the method of moments (Gibson, 2021), finite element method (Jin, 2015) and the finite-difference time-domain method (Taflove et al., 2005), etc, have been widely developed with analytically numerical methods in terms of both integral equations and differential equations to analyze antenna performance with high accuracy, which have promoted booming of EM simulators (Grout et al., 2019) and become the mainstream tool of current antenna design to greatly reduce the cost and cycle.

Naturally, this led to the combination of optimization methods to cope with nonlinear parameters optimization, which facilitated the automation of antenna design and greatly reduced the reliance on engineer experience. Classical mathematical optimizers have been applied, such as gradient descent method (Zeng et al., 2023), quasi-newton method (Hassan et al., 2011) and sequential least squares programming (Gong et al., 2023). However, these local optimization methods are very sensitive to the initial design or starting point, and tend to converge to the local optimum around the initial one, even if multiple restarts cannot guarantee satisfactory results.

1.3. Evolutionary computation for antenna design

Evolutionary computation (EC), specifically evolutionary algorithms (EAs), have been widely recognized as bio-inspired meta-heuristic global optimization algorithms to cope with antenna design problems, where design parameters are normally regarded as decision variables and specification as objectives or constraints of optimization problem (see Table 1). Compared to traditional optimization methods, EAs search within the whole design space in parallel based on population, which are less sensitive to the initial design and have distinguished themselves in terms of their powerful global search ability to find more competitive near-optimal solution. Although the multimodal and nonlinear characteristics of antenna design problems sometimes have gradient disappearance or gradient explosion and make EAs easily fall into local optima, EAs still has considerable advantages over the traditional design methods. For example, EAs have shown significant improvements over traditional methods in microstrip antennas structure optimization (Deb et al., 2011) and array synthesis (Gong et al., 2023).

The primary and common EC techniques used for antenna optimization mainly include genetic algorithm (GA) (Weile and Michielssen, 1997), particle swarm optimization (PSO) (Jin and Rahmat-Samii, 2007) and differential evolution (DE) (Rocca et al., 2011; Goudos, 2017; Kurup et al., 2003) for single objective optimization as well as multiobjective evolutionary algorithm (MOEA) (Santos et al., 2020) such as nondominated sorting genetic algorithm II (NSGA-II) (Wang et al., 2019) and multiobjective evolutionary algorithm based on decomposition (MOEA/D) (Carvalho et al., 2012) for multiobjective optimization.

When constraints are involved in antenna problem formulation, EAs with constraint-handling technique (CHT) are applied (Xu et al., 2020), mainly including penalty function method, feasibility rule method and dynamic multiobjective method. Constraints in antenna design problems divide design space into feasible region and infeasible region. In

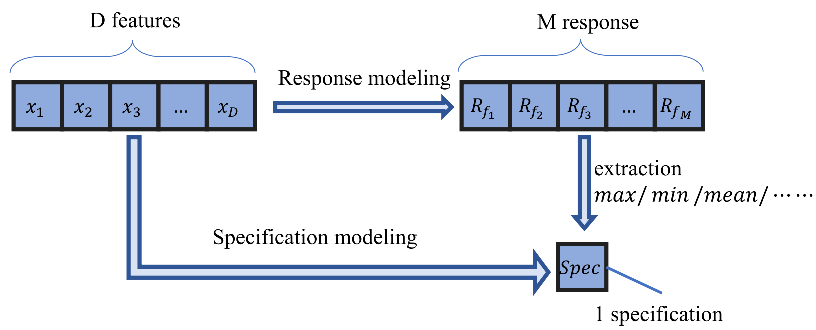


Fig. 3. Illustration of numerical difference between response modeling and specification modeling.

particular, infeasible solutions cannot be evaluated when hard constraints are involved in antenna problems, for example, violation of the minimum array spacing constraint in array design will result in simulation failure, which greatly limits the searchable space of EAs.

1.4. Machine learning assisted optimization for antenna design

Although EAs have been widely applied to antenna design, hundreds of thousands of EM evaluations are required to obtain near-optimal designs, which brings computationally expensive cost since one fullwave EM simulation is time-consuming for contemporary complex antenna design. To lower the computational overhead, machine learning (ML) methods have been incorporated to accelerate antenna optimization by building surrogate model of EM simulation from the way of expensive EM theories (called response model) and expensive objective or constrained functions (called specification model).

Specifically, ML methods, typically regression, learn from simulated or measured data with the design parameters of the antenna as feature and response/specification as label (see Table 1). Then response/specification can be predicted on unevaluated designs to replace EM fullwave simulation and further combined with EAs to locate candidate designs for EM evaluation, thereby greatly reducing computational cost. ML have shown significant improvements over traditional methods with a 3 to 7 times speed enhancement for antenna design optimization (Liu et al., 2014). Additionally, ML methods can conquer multimodal and nonlinear characteristics of antenna design problems in manner of fitness smoothing, where an easier surrogate fitness problems is approximated to guide the search. When constraints are involved in antenna specification, ML methods are further applied to approximate each constraint function. The feasible region of surrogate is determined but less reliable due to prediction uncertainty, which brings additional challenges to optimization.

The primary and common ML techniques used for antenna modeling include support vector regression (SVR) (Prado et al., 2018a,b, 2019, 2022, 2023), Gaussian process regression (GPR) (Liu et al., 2014, 2021; Jacobs and Koziel, 2013; Jacobs, 2014; Wang et al., 2021) and artificial neural networks (ANN) (Xiao et al., 2018; Gong et al., 2020; Cui et al., 2021; Budak et al., 2021; Papathanasopoulos et al., 2023).

1.5. Further illustration of two kinds of surrogate model for antenna design

To replace fullwave simulation and accelerate antenna optimization, both response and specification have been modeled with ML (Liu et al., 2021). Response modeling and specification modeling normally share the same feature description, but differ in the physical meaning of the label, where response modeling puts more emphasis on predicting EM distribution of solving Maxwell's equations and the specification modeling more on optimization. Main differences between them are discussed below.

From perspective of numerical computation (refer to Fig. 3), antenna response is vector while specification is scalar. Dimension of response normally depends on the granularity of sampling and becomes extremely high. This poses challenges that numerous models are required to predict on each dimension since traditional ML methods deal with scalar label. This also brings difficulties similar to "curse of dimensionality" to use a more complex model with many outputs that model complexity becomes excessively high and a rapidly increasing number of data are required. On the contrary, traditional ML methods can directly treat specifications as labels and get good application.

From perspective of engineering application (refer to Fig. 4), response modeling is more informative that functional relationship from angular and frequency to response is learned while specification modeling does not contain these EM knowledge. For example, antenna responses, such as S_{11} within the band of interest, radiation pattern over all interested radiation angles, are directly regarded as label in response modeling to provide more useful EM information under the same design. On the contrary, maximum of S_{11} within the band of interest, SLL, are regarded as label in specification modeling and cause information loss.

Moreover, compared to specification modeling, response modeling provides more degrees of design freedom, which can be applied to not only parameters optimization but also more complex design scenarios with high simulation expense such as redesign under different operating conditions (operating frequency and bandwidth), various material parameters (substrate permittivity and thickness), statistical analysis (e.g., yield estimation Ochoa and Cangellaris, 2013), as well as robust design (e.g., optimization accounting for manufacturing tolerances Koziel and Bandler, 2014, design centering Abdel-Malek et al., 2006).

From perspective of assisting optimization (refer to Fig. 5), response modeling (Wu et al., 2023) typically increases freedom in problem formulation in antenna design field (see Fig. 5(a)) while specification modeling (Liu et al., 2014) fails to decouple expensive costs from problem formulation (see Fig. 5(b)). Specifically, response modeling builds response model initially and specifications are calculated from response prediction at a very low cost, which makes it possible to formulate antenna optimization problems in different stage. On the contrary, specification modeling builds specification model directly, where specification model output is consistent with objectives and constraints of optimization problem so that it cannot cope with multiple antenna problem formulation.

1.6. Motivation of this work

Relation between ML and EC for antenna design is illustrated in Fig. 6 that the two are distinct but cooperative. Some surveys have provided detailed introduction of ML (Wu et al., 2020a; Akinsolu et al., 2020; El Misilmani and Naous, 2019; El Misilmani et al., 2020) (mainly focus on red circle area without distinguishing between response and specification) and EC (Weile and Michielssen, 1997; Hoorfar, 2007; Goudos et al., 2016) (mainly focus on blue shaded area) for antenna design. However, existing surveys mainly focus on one or the other AI applications, and have overlooked differences between response modeling and specification modeling.

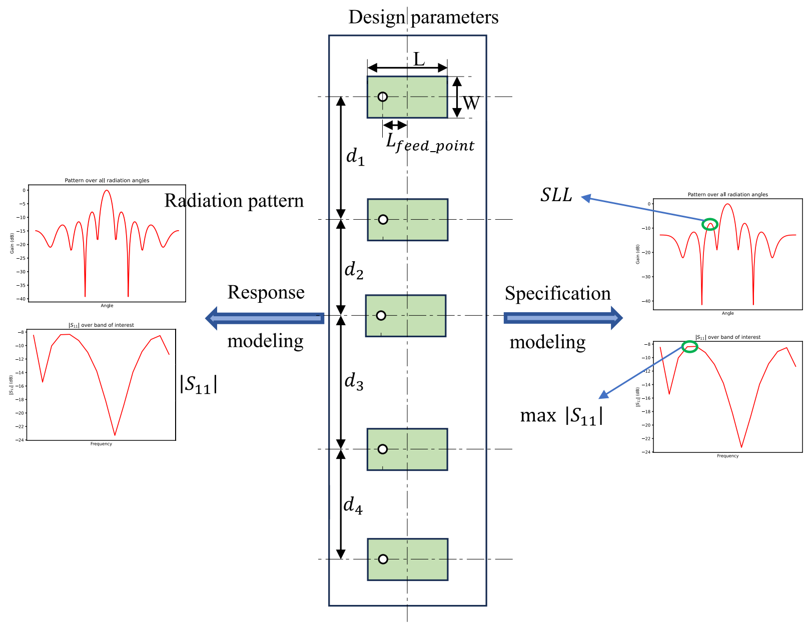
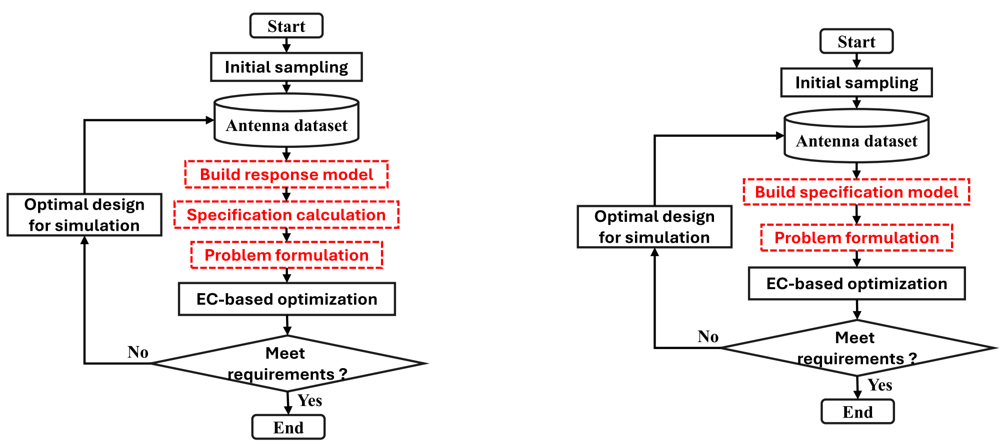


Fig. 4. Illustration of engineering difference between response modeling and specification modeling.



(a) Response modeling assisted evolutionary antenna optimization

(b) Specification modeling assisted evolutionary antenna optimization

Fig. 5. Difference between response modeling and specification modeling assisted evolutionary antenna optimization.

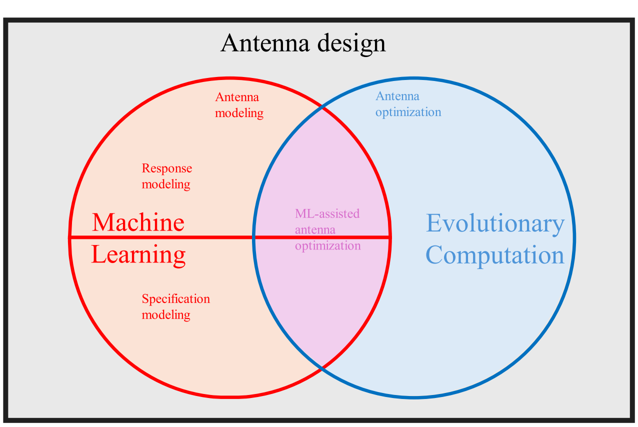


Fig. 6. Illustration of machine learning and evolutionary computation for antenna design.

1.7. Main intuition of this work

Different from these surveys, we provide a comprehensive introduction of both ML and EC for antenna design. Specifically, we review prominent methods in three parts respectively. In the first part (blue shaded area), antenna optimization with EC is discussed. According to the antenna optimization problem formulation, existing work is divided into three categories (Tables 3, 4), including single objective optimization, multi-objective optimization and constrained optimization. In the second part (red upper semicircle area), ML-assisted antenna optimization with response modeling is introduced. Methods are grouped into five categories according to how mapping from design parameters (features) to response vector (label) is learned (Tables 5, 6). In the third part (red lower semicircle area), ML-assisted antenna optimization with specification modeling is presented. Existing work is classified and discussed according to ML used (Table 7). Finally, challenges and future directions are discussed.

Table 2
List of acronyms.

Acronym	Definition
ABE	Active base element
AEP	Active element pattern
AI	Artificial intelligence
ANN	Artificial neural networks
CHT	Constraint-handling technique
CMOP	Constrained multiobjective optimization problem
COP	Constrained optimization problem
DE	Differential evolution
DFT	Discrete Fourier transform
EA	Evolutionary algorithms
EC	Evolutionary computation
EM	Electromagnetic
GA	Genetic algorithm
GF	Gaussian function
GPR	Gaussian process regression
IDFT	Inverse discrete Fourier transform
KNN	K-nearest neighbor
LCB	Lower confidence bound
ML	Machine learning
MLP	Multilayer perceptron
MOEA	Multiobjective evolutionary algorithms
MOEA/D	Multiobjective evolutionary algorithm based on decomposition
MOP	Multiobjective optimization problem
MSE	Mean square error
NSGA-II	Nondominated sorting genetic algorithm II
PSO	Particle swarm optimization
RBF	Radial basis function
RMSE	Root mean square error
S_{11}	S-parameter of reflection coefficient
SLL	Side-lobe level
SOP	Single optimization problem
SVR	Support vector regression
TF	Transfer function
VSWR	Voltage standing wave ratio

1.8. Contribution and rest structure of this work

The main contributions of this paper are as follows:

1. We provide a comprehensive summary of both ML and EC applied to antenna design. We hope that this survey delivers a systematic understanding of difficulties, methods, application and challenges of these two AI approaches.
2. Differences between response modeling and specification modeling are highlighted. We present a new taxonomy of antenna response modeling in terms of how complex mapping relationships with overlong label vector are learned by ML methods. This is critical for theoretical research of ML methods as well as specific engineering analyses of complex antenna.
3. We present an in-depth analysis of challenges of AI-empowered antenna design, and suggest promising research directions.

The rest structure of this paper is organized as follows. Section 2 briefly introduces commonly used ML and EC methods. Section 3 summarizes evolved antenna in terms of optimization problem formulation. Section 4 provides a new taxonomy of existing response modeling methods for antenna optimization. Section 5 outlines the mainstream specification modeling for accelerating antenna optimization. Section 6 presents detailed discussions of existing challenges and suggests promising future research directions. Finally, Section 7 concludes the paper. Notably, acronyms used in this paper are listed in Table 2.

2. Overview of related artificial intelligence techniques

To make it easier for readers who have less knowledge of AI techniques and make it convenient for following discussion, several most widely used ML and EC methods in literatures are briefly introduced. Due to space limitations, other advanced related methods are not included in this section, but introduce them directly along with the corresponding citations.

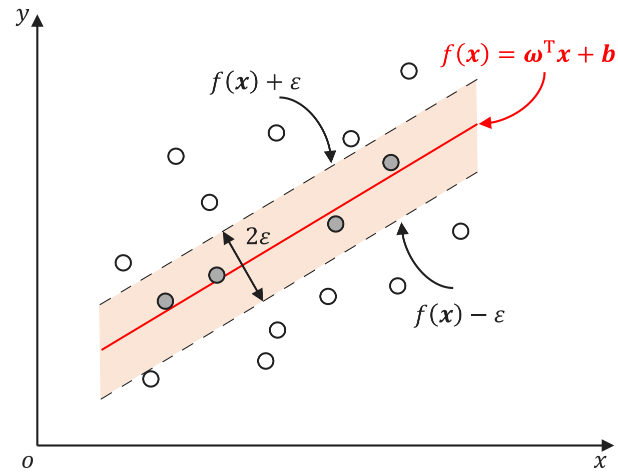


Fig. 7. Illustration of support vector regression.

2.1. Machine learning

Three commonly used ML methods are introduced, including SVR, GPR and ANN. They have been widely applied to build surrogate model of EM simulation, such as gain, S_{11} and so on.

2.1.1. Support vector regression

SVR is developed from statistical learning theory (Smola and Schölkopf, 2004). Main intuition is illustrated in Fig. 7. To map features into label, SVR builds a band structure with width of 2ε centered on linear model $f(\boldsymbol{x}) = \boldsymbol{\omega}^T\boldsymbol{x} + b$. Only when training data falls outside this interval, the loss is calculated. Its particularity lies in minimizing the ε insensitive loss while maximizing the margin, which specializes preference to smoother ones and avoids overfitting.

To find $\boldsymbol{\omega}$ and b of linear model that minimize original problem, it is normally transformed into equivalent dual problem that slack variables and Lagrange multipliers are leveraged. To deal with nonlinear problems, kernel trick is introduced. Firstly, low-dimensional input space is converted into high-dimensional feature space through feature mapping $\boldsymbol{\phi}$, and the regression can be accurately performed using a linear function $f(\boldsymbol{x}) = \boldsymbol{\omega}^T\boldsymbol{\phi}(\boldsymbol{x}) + b$. Secondly, Kernel function $\kappa\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right)$ is further introduced to equate the inner product of feature mappings $\boldsymbol{\phi}(\boldsymbol{x}_i)^T\boldsymbol{\phi}(\boldsymbol{x}_j)$ with the analytic function under low-dimensional input space, which constitutes dual problems objective and prediction. Gaussian kernel function is commonly used. Once training is done, prediction on new point $\boldsymbol{x}$ is obtained according to Eq. (1).

$$\hat{f}(\boldsymbol{x}) = \sum_{i=1}^{K}(\hat{\alpha}_i - \alpha_i)\kappa\left(\boldsymbol{x}, \boldsymbol{x}_i\right) + b \quad (1)$$

where prediction consists of a linear combination of kernel functions, α and $\hat{\alpha}$ are Lagrangian multipliers optimized from dual problems, K is size of dataset, b is derived from α and $\hat{\alpha}$.

In the antenna design field, SVR has been introduced to model both antenna elements (Angiulli et al., 2007) and arrays (Zheng et al., 2011). Resonant frequency, gain, directivity, and radiation efficiency of slotted microstrip antennas were modeled with SVR (Roy et al., 2017). More recently, SVR was applied to a multiresonant unit cell in a geometrical 4-D parallelotope domain in a reflectarray antenna design (Prado et al., 2023). A search in the Web of Science database shows that there are 165 publications related to SVR and antenna from 2004 to 2024. The trend has been increasing year by year, and there has been a significant increase since 2019.

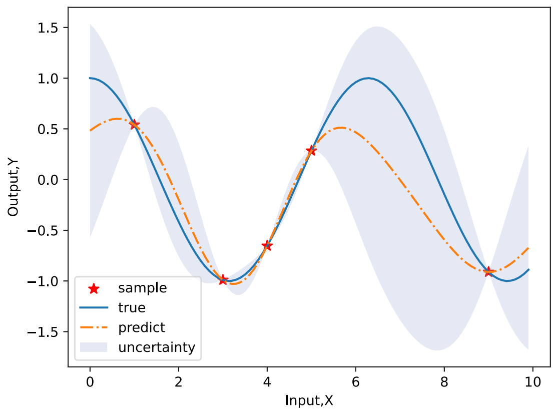


Fig. 8. An example of Gaussian process regression.

2.1.2. Gaussian process regression

GPR is statistical model that regards dataset as a sample of Gaussian stochastic process, which raises much attraction due to its ability to provide uncertainty of prediction (Santner et al., 2003). Maximum likelihood estimation is used to explain the existing observations that Gaussian joint probability density function adopts of all observations is taken as likelihood function. Kernel functions or correlation functions are introduced to measure correlation between observations, where squared exponential function Eq. (2) is mostly used.

$$c(\boldsymbol{x}, \boldsymbol{x}' \mid \boldsymbol{\theta}) = \exp(-\sum_{i=1}^{D} \theta_i |x_i - x_i'|^2) \tag{2}$$

where $\boldsymbol{\theta}$ is hyperparameter to be estimated, D is size of dimension.

To infer predictive distribution at new point $\boldsymbol{x}$, posterior distribution is obtained by best linear unbiased predictor. Predicted mean Eq. (3) and variance Eq. (4) are adjusted according to priori and correlation of existing observations with the new one.

$$\hat{f}(\boldsymbol{x}) = \hat{\mu} + \boldsymbol{r}^T \boldsymbol{C}^{-1}(\boldsymbol{y} - \boldsymbol{1}\hat{\mu}) \tag{3}$$

$$\hat{s}(\boldsymbol{x})^2 = \hat{\sigma}^2[1 - \boldsymbol{r}^{\mathrm{T}} \boldsymbol{C}^{-1} \boldsymbol{r} + \frac{(1 - \boldsymbol{1}^{\mathrm{T}} \boldsymbol{C}^{-1} \boldsymbol{r})^2}{\boldsymbol{1}^{\mathrm{T}} \boldsymbol{C}^{-1} \boldsymbol{1}}] \tag{4}$$

where $\boldsymbol{y} = (y_1, y_2, \dots, y_K)^T$ is observation vector, K is size of dataset, $\boldsymbol{C}$ is a $K \times K$ correlation matrix whose (i,j)-element is $c(\boldsymbol{x}_i, \boldsymbol{x}_j)$, $\boldsymbol{1}$ is K-dimensional column vector of ones, $\boldsymbol{r} = (c(\boldsymbol{x}, \boldsymbol{x}_1), c(\boldsymbol{x}, \boldsymbol{x}_2), \dots, c(\boldsymbol{x}, \boldsymbol{x}_K))^T$, $\hat{\mu}$ and $\hat{\sigma}^2$ are estimation of priori mean and variance.

An example of Gaussian process regression is shown in Fig. 8. The solid line represents the true function, the star points represents the samples, the dashed line represents the GPR model prediction, and the shaded area represents the uncertainty of the prediction.

In the antenna design field, GPR has been widely known as surrogate model of EM simulation to antenna design optimization (Liu et al., 2014). It is also used to accurately model the resonant frequencies of dual-band microstrip antennas (Jacobs, 2014). More recently, GPR was applied to design non-uniform metasurface circularly polarized patch antenna (Zeng et al., 2024), where input impedance bandwidth, axial ratio bandwidth and gain at boresight are considered as the optimizing targets. A search in the Web of Science database shows that there are 1145 publications related to GPR and antenna from 1983 to 2024. The trend has been increasing year by year, and there has been a significant increase since 2013.

2.1.3. Artificial neural networks

ANN are one of the most efficient learners, which are inspired by the biological brain and very well known in the name of deep learning nowadays (Goodfellow et al., 2016). An example of three-layer neural network structure is shown in Fig. 9. Features are presented at the input layer and labels are presented at the output layer. The relationship between them can be expressed as Eq. (5).

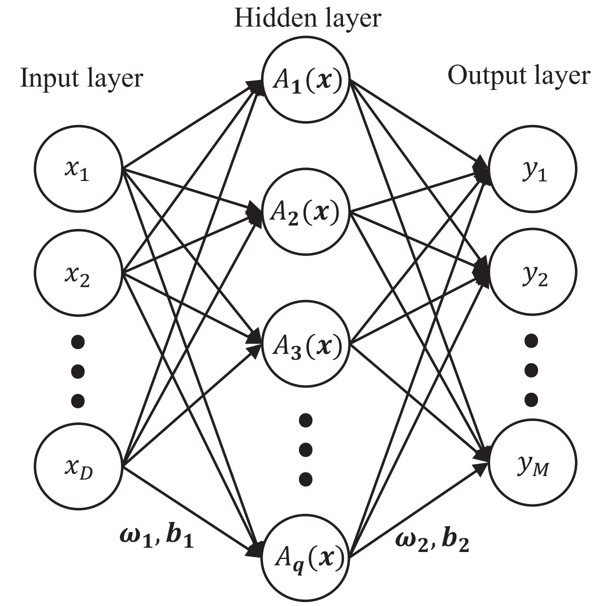


Fig. 9. A three-layer neural network structure.

$$\hat{f}(\boldsymbol{x}) = \boldsymbol{A}\left(\sum_{i=1}^{q} \omega_i A_i(\boldsymbol{x}) + b_i\right) \tag{5}$$

where hidden layer with size q is introduced to connect visible layers and extract key information from the previous layer by links between neurons, including linear combination with connection weight $\boldsymbol{\omega}$, bias $\boldsymbol{b}$ and activation function $A(\cdot)$ (rectified linear unit function, sigmoid function, etc.). Numerous hidden layers with deep structure have been proven able to deal with complex non-linear problems.

To train ANN model, mean square error (MSE) function Eq. (6) is normally taken as loss function.

$$J(\boldsymbol{\omega}, \boldsymbol{b}) = \frac{1}{K}\sum_{i=1}^{K}(\hat{y}_i - y_i)^2 \tag{6}$$

where K is the size of dataset. Note that size of output layer M is equal or greater than one, which means that ANN can be directly applied to both response modeling and specification modeling. To minimize loss function, error backpropagation algorithm, stochastic gradient descent algorithm and adaptive moment estimation algorithm are commonly used.

The quintessential example of ANN is multilayer perceptron (MLP), which is a feedforward neural network composed of multilayers and has been widely applied to antenna design.

In the antenna design field, ANN has been applied to model antenna elements (Xiao et al., 2018). It is also used to accurately model far-field pattern for array synthesis (Cui et al., 2021). More recently, ANN was applied to pattern synthesis of conformal arrays (Sun et al., 2024). A search in the Web of Science database shows that there are 3226 publications related to ANN and antenna from 1990 to 2024. The trend has been increasing year by year, and there has been a significant increase since 2018.

2.2. Evolutionary computation

EC includes single objective EAs and MOEAs. They have been applied to deal with different optimization formulation of antenna design problems.

2.2.1. Genetic algorithm

GA is inspired from theory of heredity and evolution, which can solve both continuous and combinatorial optimization problems through real value and binary encoding (Deb and Beyer, 2001).

It mainly consists of three genetic operations, including environmental selection, chromosome crossover and gene mutation. The selection operator chooses individuals as parents to generate offsprings, where roulette wheel selection and tournament selection are commonly used. The crossover operator exchanges information of parents solutions to produce child solutions, such as one-point crossover and simulated binary crossover. The mutation operator makes variation on each gene with a certain probability, which contributes to maintain diversity of population.

In the antenna design field, GA has been applied to design thinned arrays in the early stage (Haupt, 1994). It is also applied to automated synthesis of a lunar satellite antenna system (Lohn et al., 2015). More recently, GA was applied to design low sidelobe planar electrically large sparse array antenna with element number reduction (Zhu et al., 2024a). A search in the Web of Science database shows that there are 3234 publications related to GA and antenna from 1991 to 2024. The trend has been increasing year by year, and there has been a significant increase since 2003 and keep growing.

2.2.2. Particle swarm optimization

PSO (Li et al., 2011) is inspired from birds flock's foraging behavior. Each particle $\boldsymbol{x}_i$ updates its velocity $\boldsymbol{v}_i$ based on current best positions information $\boldsymbol{p}_{i\,best}$ found by itself and $\boldsymbol{g}_{best}$ by whole swarm, which contribute to exploitation and exploration respectively. Then each particle updates its position according to updated velocity. The velocity and position is updated as Eqs. (7) and (8).

$$\boldsymbol{v}_i = \omega\boldsymbol{v}_i + \eta_1 r_1(\boldsymbol{p}_{i\,best} - \boldsymbol{x}_i) + \eta_2 r_2(\boldsymbol{g}_{best} - \boldsymbol{x}_i) \tag{7}$$

$$\boldsymbol{x}_i = \boldsymbol{x}_i + \boldsymbol{v}_i \tag{8}$$

where $\omega \in (0,1)$ is inertia weight, η_1 and η_2 are acceleration constants, r_1 and r_2 are random numbers within $[0,1]$. Before next generation, $\boldsymbol{p}_{i\,best}$ and $\boldsymbol{g}_{best}$ are updated.

In the antenna design field, PSO has been applied to design ultrawideband planar antenna in the early stage (Lizzi et al., 2007). It is also applied to synthesis of unequally spaced antenna arrays (Bhattacharya et al., 2012). More recently, PSO was combined with characteristic point method to design multiband antennas (Koziel and Pietrenko-Dabrowska, 2024). A search in the Web of Science database shows that there are 2006 publications related to PSO and antenna from 2003 to 2024. The trend has been increasing year by year, and there has been a significant increase since 2008 and keep growing.

2.2.3. Differential evolution

DE uses the differences between solutions for mutation (Das and Suganthan, 2010). For each target vectors $\boldsymbol{x}_i$, mutant vector $\boldsymbol{v}_i$ is first produced. $DE/rand/1/bin$ is as follows.

$$\boldsymbol{v}_i = \boldsymbol{x}_{r1} + F(\boldsymbol{x}_{r2} - \boldsymbol{x}_{r3}) \tag{9}$$

where $r1$, $r2$, $r3$ are different indexes randomly picked from $\{1, 2, \ldots, NP\}$, NP is population size, $F \in [0,2]$ is scaling factor.

Then trial vector $\boldsymbol{u}_i$ is obtained by binomial crossover Eq. (10) with above mutant vector $\boldsymbol{v}_i$ and target vector $\boldsymbol{x}_i$.

$$u_{ij} = \begin{cases} v_{ij}, & if(rand \le CR) | j = j_{rand} \\ x_{ij}, & otherwise \end{cases} \tag{10}$$

where $j = 1, \ldots, D$, D is size of dimension, $CR \in [0,1]$ is crossover rate, j_{rand} is a randomly selected index from $\{1, \ldots, D\}$.

Finally, greedy criterion is adopted for environmental selection between target vector and trial vector as Eq. (11).

$$\boldsymbol{x}_{i,G+1} = \begin{cases} \boldsymbol{u}_{i,G} & \text{if } f(\boldsymbol{u}_{i,G}) \le f(\boldsymbol{x}_{i,G}) \\ \boldsymbol{x}_{i,G} & \text{if } f(\boldsymbol{u}_{i,G}) > f(\boldsymbol{x}_{i,G}) \end{cases} \tag{11}$$

where $f(\cdot)$ is fitness function, G is the number of current generation.

In the antenna design field, DE has been applied to design ultrawideband planar antenna in the early stage (Lizzi et al., 2007). It is also applied to synthesis of unequally spaced antenna arrays (Bhattacharya et al., 2012). More recently, DE was applied to design dielectric resonator antenna arrays (Suman et al., 2024). A search in the Web of Science database shows that there are 776 publications related to DE and antenna from 1992 to 2024. The trend has been increasing year by year, and there has been a significant increase since 2010.

2.2.4. Multiobjective evolutionary algorithm

A. NSGA-II

NSGA-II (Deb et al., 2002) is one of most famous domination-based MOEAs. After generation of child population, elitism scheme is first introduced to select next parent population that current parent population and child population are merged and nondominated elitist are reserved. To sort the combined population, fast nondominated sorting approach is then introduced and multiple nondominated layers are thus obtained. Individuals in the same layer are nondominated to each other, while individuals in the layer with lower rank dominate those with higher rank. To make a fixed size of next population and preserve diversity of population in objective space, crowding distance is introduced to sort the layer that exactly fulfill next parent population, and top individuals with size of vacancy are selected.

In the antenna design field, NSGA-II has been widely applied to synthesis of conformal phased array (Yang et al., 2009) and beamforming (Jayaprakasam et al., 2017). More recently, NSGA-II was applied to design which needs minimizing the beamwidth of the antenna mainbeam while maximizing the peak-to-sidelobe level and directivity (Wolff and Nanzer, 2024). A search in the Web of Science database shows that there are 92 publications related to NSGA-II and antenna from 2006 to 2024.

B. MOEA/D

MOEA/D (Zhang and Li, 2007) is one of most famous decomposition-based MOEAs. A multiobjective optimization problem is first decomposed into multiple scalar subproblems by a set of evenly distributed weight vectors and mathematical programming methods (weighted sum approach, Tchebycheff approach and penalty-based boundary intersection approach, etc.). Then these subproblems are optimized simultaneously by a population. The size of population is set to the number of subproblems. Each individual is assigned to one weight vector, and reproduction and selection are carried out among solutions associated with neighbor weight vectors. An external population is introduced in the original version to store nondominated solutions found so far.

In the antenna design field, MOEA/D has been applied to design a quad-band double-sided bow-tie antenna (Ding and Wang, 2013) and Yagi-Uda antennas (Carvalho et al., 2012). More recently, MOEA/D was applied to design a single-band and dual-band MIMO antenna for semantic-based mobile system (Suman et al., 2024). A search in the Web of Science database shows that there are 51 publications related to MOEA/D and antenna from 2011 to 2024.

3. Antenna optimization with evolutionary computation

3.1. Motivation

Antenna design is normally formulated as optimization problem, including single-objective optimization problem (SOP), multiobjective optimization problem (MOP), constrained optimization problem (COP) and constrained multiobjective optimization problem (CMOP) (Xu et al., 2020). Different formulations need to be solved by different types of EAs. A taxonomy of evolved antenna is shown in Fig. 10. To highlight the differences between different problem formulations and have a comprehensive view of the application of various EAs, evolved

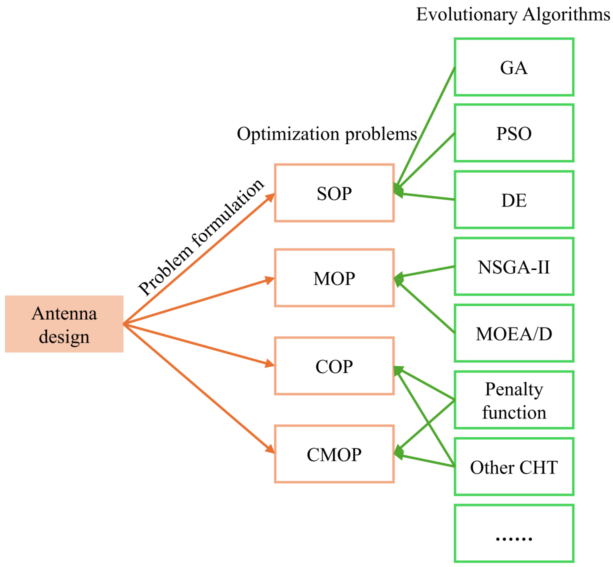


Fig. 10. Taxonomy of evolved antenna.

antennas are divided into three parts and discussed respectively, including single-objective optimization, multi-objective optimization and constrained optimization.

A general CMOP with m objectives and k constraints is taken as example and expressed as Eq. (12).

$$\begin{aligned} \text{min} \quad & \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), \dots, f_m(\boldsymbol{x})) \\ \text{st}: \quad & \boldsymbol{g}(\boldsymbol{x}) = (g_1(\boldsymbol{x}), g_2(\boldsymbol{x}), \dots, g_k(\boldsymbol{x})) \leq \boldsymbol{0} \\ \text{where} \quad & \boldsymbol{x} = (x_1, x_2, \dots, x_D) \in \mathbf{X} \\ & \mathbf{X} = \{\boldsymbol{x} | \boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u}\} \\ & \boldsymbol{l} = (l_1, l_2, \dots, l_D), \boldsymbol{u} = (u_1, u_2, \dots, u_D), \end{aligned} \tag{12}$$

where $\boldsymbol{x}$ is the solution with size D, $\mathbf{X}$ is the solution space, $\boldsymbol{l}$ and $\boldsymbol{u}$ are the lower bound and upper bound of the solution space, $\boldsymbol{f}(\cdot)$ is a vector of objective functions, $\boldsymbol{g}(\cdot) \leq \boldsymbol{0}$ is a vector of constraints.

Inclusion relationships among SOP, MOP, COP and CMOP are shown in Fig. 11 that SOP, MOP and COP can be seen as degradation cases of CMOP that CMOP degrades into MOP when $k = 0$ and into COP when $m = 1$, MOP degrades into SOP when $m = 1$, and COP degrades into SOP when $k = 0$.

3.2. Methods

3.2.1. Single-objective optimization

In this case, antenna design is formulated into SOP with objective number $m = 1$ and constraint number $k = 0$ in Eq. (12). Single-objective EAs are then applied.

A. Antennas optimized by GA

Haupt (1994) applied GA to optimize SLL of thinned arrays. Hornby et al. (2011) applied GA to optimize gain of wire antenna for NASA's space technology 5 mission. GA has also been integrated in automated design system for NASA's LADEE mission (Lohn et al., 2015). GA has also been applied to design monopole antenna (Altshuler and Linden, 1997), Yagi-Uda antenna (Jones and Joines, 1997) and miniaturized meander-line antennas (Marrocco et al., 2002).

B. Antennas optimized by PSO

Lizzi et al. (2007) applied PSO to optimize return loss of a spline-shaped ultrawideband antenna. Goudos et al. (2010) applied comprehensive learning PSO to synthesize unequally spaced linear array, where SLL, beamwidth and null are formulated into single objective. Bhattacharya et al. (2012) applied position mutated hierarchical PSO to synthesize unequally spaced linear array. Li et al. (2013) applied PSO with neighborhood redispatch technique to optimize VSWR of an ubtrawideband antenna.

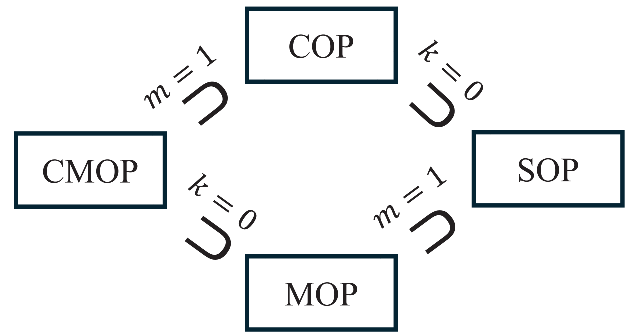


Fig. 11. Relation among different optimization problems.

C. Antennas optimized by DE

Deb et al. (2011) applied DE to optimize reflection coefficient of aperture coupled microstrip antennas. Montgomery et al. (2011) extended DE to design radio frequency identification antennas with discrete variable. Ma et al. (2019) applied DE to synthesize irregular arrays and achieved excellent beam scanning ability.

D. Antennas optimized by other single-objective EAs

In addition to the above three most commonly used methods, several other evolutionary optimization methods have also been applied. For example, genetic programming was applied to design wire antenna (Lohn et al., 2005). Evolutionary programming was applied to design broadband parasitic wire arrays (Casula et al., 2011). Gregory et al. (2011) applied covariance matrix adaptation evolutionary strategy to design wideband stacked-patch antenna and ultrawideband array.

Multiple EAs have been hybridized to design antenna. Li et al. (2010) developed a hybrid EA of GA and PSO to synthesize conformal array pattern. Yang et al. (2013) developed a hybrid EA of artificial bee colony and DE to synthesize time-modulated arrays pattern. Grimaccia et al. (2007) developed a hybrid EA that combined GA and PSO to generate new population to optimize phased array.

EAs and mathematical methods have also been hybridized to design antenna. Yang et al. (2017) combined convex programming and DE to synthesize heterogeneous arrays. Cui et al. (2017) combined GA and modified iterative Fourier transform to synthesize thinned array. Gong et al. (2023) combined DE and sequential least squares programming to synthesize linear arrays.

3.2.2. Multi-objective optimization

In this case, antenna design is formulated into MOP with objective number $m > 1$ and constraint number $k = 0$ in Eq. (12). MOEAs are then applied.

A. Antennas optimized by NSGA-II

Kuwahara (2005) applied Pareto GA to optimize Yagi–Uda antenna, where gain, SLL, and VSWR are set as objectives. Goudos et al. (2013) applied NSGA-II with local search to the synthesis of uniform and nonuniform subarrayed linear arrays, where objectives include directivity maximization and SLL minimization. Wang et al. (2019) applied NSGA-II to design a multiband pixel patch antenna, where gain and bandwidth are set as objectives. Yuan et al. (2012) applied NSGA-II to design a parasitic layer-based reconfigurable antenna, where three objectives are gain, bandwidth and axial ratio. Improved NSGA-II has also been applied to array synthesis (Jayaprakasam et al., 2017; Yang et al., 2009), where direction of the beam peak, SLL and beamwidth are considered.

B. Antennas optimized by MOEA/D

Liu et al. (2012) applied multiple-single-objective Pareto sampling algorithm to optimize linear array, where five objectives are formulated. Carvalho et al. (2012) applied MOEA/D to design broadband optimal Yagi-Uda antennas and three objectives are directivity, VSWR and front-to-back ratio. Ding and Wang (2013) applied a modified MOEA/D to design a tri-band bow-tie antenna, where return loss in each band is set as objective respectively. Lu et al. (2017) applied MOEA/D with genetic operator to optimize isolation of MIMO antennas, and further applied MOEA/D-DE to optimize return loss. Li et al. (2019) developed a new MOEA named MOEA/D-GPSO by embedding PSO into MOEA/D to optimize compact log-periodic dipole array, where objectives are VSWR, gain and total length. Li et al. (2020) proposed an improved MOEA named MOEA/D-M by integrating PSO and binary PSO into MOEA/D to optimize isolation and return loss of compact MIMO antenna, which deals with continuous and discrete parameters simultaneously.

3.2.3. Constrained optimization

In this case, antenna design is formulated into COP or CMOP that constraint number $k > 0$ in Eq. (12). EAs with constraint-handling technique are then applied.

A. Handling constraints via penalty function

Penalty function method incorporates a penalty term into the objective function via penalty value to penalize the violation of constraints.

Jamnejad and Hoorfar (2004) applied evolutionary programming to design corrugated circular horn, where pattern is set as objective function and penalized by constraints of return loss, beam width, pattern circularity and low cross-polarization.

B. Handling constraints via feasibility rule

The feasibility rule method ensures the generation of feasible solutions since solutions with smaller violation survive. Objective value becomes the second evaluation criterion.

Cai et al. (2008) combined this method with GA to design an X-band antenna for NASA's space technology 5 spacecraft, where gain is set as objectives and VSWR at the transmit and receive frequencies as constraints. Liu et al. (2013) combined this method with DE to design a wideband twisted dipole antenna, where objective and constraint are formulated as return loss.

C. Handling constraints via dynamic multiobjective method

Dynamic multiobjective method convert constraint violation to an additional objective and MOEA is applied, which is indicated as DC-MOEA. To have a better balance between objective and constraint, violation is dynamically reduced.

Dong et al. (2014) applied this method to design a linear sparse arrays, where objectives and constraints are formulated with SLL, beamwidth and spatial aperture. Jiang et al. (2016) applied this method to design a wide-band helical antenna, where right-hand circular polarization gain and height of the antenna are set as two objectives, gain, VSWR and axial ratio are set as constraints.

3.3. Summary

To summarize applications of EC for antenna design, we investigate typical research of evolved antenna in Table 3 and make a comprehensive summary in terms of antenna type, problem and algorithm.

In addition to the above applications, EC has also been applied in some other special cases involved in specific engineering practice (parallelization, robustness, eg.). For example, PSO for antenna design is implemented on parallel clusters to reduce computational time (Jin and Rahmat-Samii, 2005). To obtain robust antenna design, variance of gain, VSWR and axial ratio over frequency band is optimized (Hu et al., 2019). To achieve reliable antenna design, worst-case sensitivity analysis (Zhang and Rahmat-Samii, 2017) and the influence of uncertainties (Steiner et al., 2004) have also been investigated.

3.4. Remarks

Based on above discussion, we comment on highlights and limitations of different antenna problem formulations in Table 4.

4. Machine learning assisted antenna optimization with response modeling

4.1. Motivation

Response modeling have raised much attention in literatures. To highlight their differences to specification modeling and present essential difficulties of the problem, we categorize existing methods that how ML methods are introduced to learn mapping f from antenna design parameters $\boldsymbol{x}$ to response vector $\boldsymbol{R}$ as Eq. (13) with given training set $\{(\boldsymbol{x}_1, \boldsymbol{R}_1), (\boldsymbol{x}_2, \boldsymbol{R}_2), \dots, (\boldsymbol{x}_K, \boldsymbol{R}_K)\}$.

$$\underset{(x_1, x_2, \dots, x_D)^T}{\boldsymbol{x}} \xrightarrow{f} \underset{(R_1, R_2, \dots, R_M)^T}{\boldsymbol{R}} \tag{13}$$

where D is size of features, M is size of label (sampling granularity of frequency for S_{11} and angular for radiation pattern, eg.), K is size of dataset. Note that M is larger than one.

4.2. Methods

4.2.1. Direct modeling

The simplest way is to learn mapping f in Eq. (13) with existing ML methods directly. To be specific, one is to build multiple single-output machine learning models for each dimension response value, and the other is to build a multi-output machine learning model for all dimension response values.

A. Single-output model

In this mode, trainset is split into M subsets with scalar label, and single-output ML methods are directly applied. Total M models are fitted that $\boldsymbol{x}$ is set as feature and each dimension of response vector $\boldsymbol{R}$ is set as label by turns.

Koziel et al. (2012) applied co-kriging model to predict S_{11} response. As pointed out, model construction typically takes several minutes for 100 frequency points case. Chen et al. (2017) applied kriging model (similar to GPR) to predict S_{11} curve magnitude of E-shaped patch antenna. Frequency band 4.5 GHz∼6.5 GHz is represented with 40 discreted frequency points and kriging model is fitted on each one. They further applied response model to optimize two resonant frequencies that maximum of S_{11} at {5 GHz, 5.5 GHz} is formulated as objective and optimized by DE. Prado et al. (2019) applied SVR to predict reflection coefficients of a very large shaped-beam reflectarray for direct broadcast satellite, where real and imaginary parts are estimated separately, 5 frequency and 52 angles of incidence are considered. They extended their work to predict the electromagnetic response of the constituent unit cell for a direct layout optimization of the antenna (Prado et al., 2022), where 64 angles of incidence at a single frequency is considered and it takes 71 days to obtain 529 samples.

B. Multi-output model

In this mode, multi-output ML methods (particularly ANN with multiple output layer nodes) are directly applied. Only one model is fitted that $\boldsymbol{x}$ is set as input and response vector $\boldsymbol{R}$ is set as output.

Cui et al. (2021) applied ANN for linear array beampattern synthesis and decoder ANN is trained as array analyzer, where excitation serve as model input and far field pattern serve as output. Experiments are done on three array cases, including ideal array and actual array with elements number varying from 32 to 149. Note that output layer size of 3602 is so large that model complexity increases dramatically and at least 5000 samples is required. They extended this method to

Table 3
Summary of various evolved antennas.

References	Antenna type	Problem			Algorithm
		Formulation	Objectives	Constraints	
Haupt (1994)	Thinned arrays	SOP	SLL	–	GA
Hornby et al. (2011)	X-band wire antenna	SOP	Gain	–	GA
Jones and Joines (1997)	Yagi-Uda antenna array	SOP	Combination of gain, impedance, SLL	–	GA
Lizzi et al. (2007)	Spline-shaped ultrawideband antenna	SOP	Combination of S_{11}, S_{21}, group delay	–	PSO
Goudos et al. (2010)	Unequally spaced linear array	SOP	Combination of SLL, beamwidth, null	–	CLPSO
Li et al. (2013)	Coplanar waveguide fed ultrawideband antenna	SOP	VSWR	–	NR-PSO
Deb et al. (2011)	Aperture coupled microstrip antennas	SOP	Reflection coefficient	–	DE
Montgomery et al. (2011)	Radio frequency identification antenna	SOP	Combination of efficiency, resonant frequency	–	DE
Ma et al. (2019)	4-D irregular antenna array	SOP	Combination of SLL, directivity, efficiency	–	DE
Yuan et al. (2012)	Parasitic layer-based reconfigurable antenna	MOP	Gain, bandwidth, axial ratio	–	NSGA-II
Goudos et al. (2013)	Subarrayed linear arrays	MOP	Directivity, SLL	–	NSGA-II
Yang et al. (2009)	Conformal phased array	MOP	Beam peak direction, SLL, beamwidth	–	Improve NSGA-II
Carvalho et al. (2012)	Yagi-Uda antennas	MOP	Directivity, VSWR, front-to-back ratio	–	MOEA/D
Ding and Wang (2013)	Tri-band bow-tie antenna	MOP	S_{11} in three band	–	Modified MOEA/D
Li et al. (2019)	Compact log-periodic dipole array	MOP	VSWR, gain, total length	–	MOEA/D-GPSO
Jamnejad and Hoorfar (2004)	Corrugated horn antennas	COP	Pattern	Return loss, beamwidth, pattern circularity, cross-polarization	Evolutionary programming with penalty function
Cai et al. (2008)	X-band wire antenna	COP	Gain	VSWR at transmit and receive frequency	GA with feasibility rule
Liu et al. (2013)	Wideband twisted dipole antenna	COP	Return loss	Return loss	DE with feasibility rule
Dong et al. (2014)	Linear sparse array	CMOP	SLL, beamwidth, spatial aperture	SLL, beamwidth, spatial aperture	DCMOEA
Jiang et al. (2016)	Wideband helical antenna	CMOP	Right-hand circular polarization gain, height	Gain, VSWR, axial ratio	DCMOEA
Xu et al. (2020)	Linear array	CMOP	Beamwidth, SLL, null	Beamwidth, null	DCMOEA

Table 4
Comments on the highlights and limitations of different antenna problem formulation.

Methods	Highlights	Limitations
SOP	(1) Relatively simple. (2) A large number of algorithms for solving SOP.	(1) Could not show the trade-off between objectives. (2) Probably fail to satisfy some constraints. (3) Could not provide multiple optimal solutions. (4) The optimal solution probably does not have practical significance.
COP	(1) Fully meet multiple constraints.	(1) Could not show the trade-off between objectives. (2) Could not provide multiple optimal solutions.
MOP	(1) Well express multiple objectives. (2) Multiple optimal solutions.	(1) Probably fail to satisfy some constraints.
CMOP	(1) Fully express multiple objectives. (2) Ensure multiple constraints. (3) A mass of well-representative Pareto-optima.	(1) Few algorithms for addressing CMOP.

synthesis of pattern-reconfigurable array (Cui et al., 2022) and time-modulated arrays (Hei et al., 2023). Gong and Xiao (2019) proposed to predict active element pattern (AEP) by ANN so that mutual coupling effects are considered for synthesis. Model has 2896 outputs and thousands of samples are required. They extended this work to deal with S parameters simultaneously (Gong et al., 2020) and planar array cases (Gong et al., 2021). Jiao et al. (2023) optimized mmWave array by three stages, where ANN is applied to predict S parameters curve and radiation pattern at all frequency points respectively.

4.2.2. Parametric modeling

Parametric modeling methods transform label vector $\boldsymbol{R}$ to low dimension via a parameterized function with much lower dimension parameter vector $\boldsymbol{R_L}$ ($\boldsymbol{R_L}$ is transformed parameters and is determined by the vector $\boldsymbol{R}$), establish model with $\boldsymbol{R_L}$ as new label. When making predictions on new samples, restore $\hat{\boldsymbol{R}}$ with predicted $\hat{\boldsymbol{R_L}}$ by the $\boldsymbol{R_L}$ parameterized function.

A. Transfer function based parametric modeling

Transfer function (TF) is from circuit theory as Eq. (14).

$$H(s) = \sum_{i=1}^{Q} \frac{r_i}{s - p_i} \tag{14}$$

where p_i is the pole coefficient that contributes to represent a peak, r_i is the residue coefficients, s represents frequency, Q is the order that corresponds to the number of peaks. To parametrically model $\boldsymbol{R}$ with TF, we take S_{11} response as example as follows.

Step 1: For each response $\boldsymbol{R} = (S_{11\,freq_1}, S_{11\,freq_2}, \ldots, S_{11\,freq_M})^T$, reorganize the indexes and corresponding values in $\boldsymbol{R}$ into pairs, and we get dataset $\{(freq_i, S_{11\,freq_i})\}, i = 1, \ldots, M$.

Step 2: Use this dataset to fit TF until parameters p_i and r_i are identified, and we obtain $\boldsymbol{R_L} = (p_1, \ldots, p_Q, r_1, \ldots, r_Q)^T$ with size of $2Q$ far less than M.

Step 3: Based on transformed training set $\{(\boldsymbol{x}_1, \boldsymbol{R}_{L1}), (\boldsymbol{x}_2, \boldsymbol{R}_{L2}), \ldots, (\boldsymbol{x}_K, \boldsymbol{R}_{LK})\}$, train ML model.

Step 4: Use ML model in *Step 3* to predict $\hat{\boldsymbol{R_L}}$ at new points $\boldsymbol{x}^*$.

Step 5: For each frequency s, calculate Eq. (14) with $\hat{\boldsymbol{R_L}}$, and we obtain $\hat{\boldsymbol{R}}$ at $\boldsymbol{x}^*$.

Following rational form of TF as Eq. (15) has also been used Gustavsen and Semlyen (1999).

$$H(s) = \frac{\sum_{i=0}^{P} a_i s^i}{1 + \sum_{i=1}^{Q} b_i s^i} \tag{15}$$

where a_i and b_i are parameters to be determined, corresponding $\boldsymbol{R_L} = (a_1, \ldots, a_P, b_1, \ldots, b_Q)^T$ with size of $P + Q$. Note that parameterization process needs to be carried out independently for each sample label and the order of the TF needs to be predefined.

Cao et al. (2009) first combined ANN and TF to model response. Feng et al. (2015) developed a pole-residue tracking technique to solve order-changing problem that some predefined order is not sufficient to obtain good fitting consistency and transformed parameters $\boldsymbol{R_L}$ have different sizes. Xiao et al. (2017) applied TF to parameterize S_{11} response and then used ANN to learn TF coefficients. As for order-changing problem, support vector machine was applied to classify samples according to TF orders and ANN model was built for each cluster respectively. They extended this work to multiparameter modeling (Xiao et al., 2018), where gain and radiation pattern were considered and frequency s was replaced with angle θ in the parameterization of the radiation pattern. They also extended their work to deal with finite periodic arrays (Xiao et al., 2019), thinned arrays (Xiao et al., 2020) and AEP modeling (Hong et al., 2020, 2022). Chen et al. (2020) applied TF and ANN to model AEP of five-element unequally spaced linear array. Ma et al. (2023) applied TF and ANN to model reflection coefficients of frequency selective structure. Luo et al. (2020) combined convolutional neural networks and TF to predict response, where antenna structure was replaced by antenna image.

B. Gaussian function based parametric modeling

Gaussian function (GF) based parametric modeling is same as TF based parametric modeling, only different that Gaussian function as Eq. (16) is used.

$$G(s) = \sum_{i=1}^{Q} c_i e^{-\frac{(s-b_i)^2}{a_i}} + d \tag{16}$$

where a_i, b_i, c_i and d are coefficients, s is frequency, Q is number of basis. Consequently, $\boldsymbol{R_L} = (a_1, \ldots, a_Q, b_1, \ldots, b_Q, c_1, \ldots, c_Q, d)^T$ with size of $3Q + 1$. Gaussian function is simple and relatively insensitive to parameter errors. Note that Q is usually taken as the number of peaks of the response to be fitted, and its setting highly depends on the experience of the engineers.

Kim et al. (2007) applied GF and MLP to model the impedance of wideband antenna. Q is set to six that impedance curve has six hills and total 19 parameters form $\boldsymbol{R_L}$. They further applied this model to optimize average VSWR over frequency range and saved much time.

C. Discrete Fourier transform based parametric modeling

The reason why discrete Fourier transform (DFT) is used to parametrically model response is as follows. Since array factor and the inverse discrete Fourier transform (IDFT) have a very high similarity in the form of the formulas, it is assumed that element excitations of a periodic spacing array can be obtained by DFT of corresponding AF, which is described as Eq. (17).

$$AF(n) = \sum_{k=0}^{NE-1} a(k) e^{jkn}, n = \frac{2\pi}{\lambda} d \sin\psi \tag{17}$$

where NE is the number of elements, $a(k)$ is the kth excitation coefficient, d is the element spacing and ψ is the inclination angle to the normal of the array.

Based on above theory assumptions, to model radiation pattern, procedures are as follows.

Step 1: For each response $\boldsymbol{R}$, a DFT is performed and we obtain $\boldsymbol{R_L} = (a_0, a_1, \ldots, a_{NE-1})^T$ that have the physical meaning of excitation.

Step 2: Based on transformed training set $\{(\boldsymbol{x}_1, \boldsymbol{R}_{L1}), (\boldsymbol{x}_2, \boldsymbol{R}_{L2}), \ldots, (\boldsymbol{x}_K, \boldsymbol{R}_{LK})\}$, train ML model.

Step 3: Use ML model in *Step 2* to predict $\hat{\boldsymbol{R_L}}$ at new points $\boldsymbol{x}^*$.

Step 4: For each $\hat{\boldsymbol{R_L}}$, perform IDFT as Eq. (17), and we obtain $\hat{\boldsymbol{R}}$ at $\boldsymbol{x}^*$.

Note that NE is predefined according that reconstruction error between simulated and calculated results is below a certain threshold.

Wu et al. (2023) combined this method and kriging model to predict radiation pattern. They also integrated this parametric model into surrogate-assisted optimization framework with NSGA-II as optimizer.

4.2.3. Additional feature modeling

Extract the index in the response $\boldsymbol{R}$ as an additional feature (usually frequency for S_{11} and angle for radiation pattern), build ML models with additional feature so that "curse of dimensionality" in label is avoided. Take S_{11} as example, specific steps are as follows. Note that frequency is replaced with angle for radiation pattern modeling.

Step 1: For each sample $(\boldsymbol{x}, \boldsymbol{R})$ with $\boldsymbol{R} = (S_{11\,freq_1}, S_{11\,freq_2}, \ldots, S_{11\,freq_M})^T$, take frequency as additional feature and corresponding response value as label, and we obtain M new samples with scalar label $\{((\boldsymbol{x}, freq_1), S_{11\,freq_1}), \ldots, ((\boldsymbol{x}, freq_M), S_{11\,freq_M})\}$.

Step 2: Based on transformed training set with size of $K \times M$ $\{((\boldsymbol{x_1}, freq_1), S_{11\,freq_1}), \ldots, ((\boldsymbol{x_1}, freq_M), S_{11\,freq_M}), \ldots, ((\boldsymbol{x_K}, freq_1), S_{11\,freq_1}), \ldots, ((\boldsymbol{x_K}, freq_M), S_{11\,freq_M})\}$, train ML model.

Step 3: For each frequency point, together with new points $\boldsymbol{x}^*$, use ML model in *Step 2* to get M predictions and thus obtain $\hat{\boldsymbol{R}}$.

Angiulli et al. (2007) applied SVR to model impedance, reactance, magnitude and phase curves of microwave devices and antennas. Frequency and angle were taken as additional features respectively and results showed that SVR achieved better root mean square errors (RMSE) than MLP. Jacobs (2012) applied Bayesian SVR to model S_{11} response and obtained better RMSE than SVR. They also incorporated multifidelity modeling that support vectors from coarse data were simulated with fine discretization and used to build fine model so that simulation cost is reduced (Jacobs et al., 2012). Wu et al. (2019a) proposed ML-assisted optimization with additional feature to optimize antenna parameters, where frequency is added as feature to train GPR model and maximum of lower confidence bound (LCB) among all frequency points is set as fitness. They extended their work to design wideband millimeter-wave horizontally polarized omnidirectional antenna (Gong et al., 2022), broadband reflectarray antenna (Cao et al., 2019), mmWave array (Wu et al., 2021b).

Wu et al. (2021c,a) proposed active base element (ABE) modeling for array synthesis. By modeling AEP of each element with adjacent element spacings as feature, frequency and angle as additional features, ABE model is obtained. Note that size of transformed training set is further multiplied by the number of elements since element is basic modeling object. To alleviate model computational cost with overabundant training set, virtual subarray expansion is introduced (Wu et al., 2022b). They also extended their work to design planar array (Wu et al., 2022a) and series-fed microstrip arrays (Chen et al., 2023).

Additional feature modeling is also incorporated with multifidelity modeling for data enhancement. The mapping from low fidelity data to high fidelity data was learned. Jacobs and Koziel (2013) proposed two stage modeling framework. The first step is to use GPR to learn the mapping relationship between the low-fidelity and high-fidelity S_{11} to expand the high-fidelity data set. The second step is to use the expanded data set to build GPR to predict and optimize the response. Wu et al. (2019b, 2020b) applied multioutput GPR to further learn correlations between S_{11} and gain.

4.2.4. K-nearest neighbor based modeling

K-nearest neighbor (KNN) is a neighbor-based ML method. The value of the new data is determined by how much it resembles the training set. To be specific, neighbors of new point are identified by distance (such as Manhattan distance, Euclidean distance and Chebyshev distance), and average or weighted average of neighbors label are set as prediction. To predict $\hat{\boldsymbol{R}}$ at new point $\boldsymbol{x}^*$ with KNN, the neighbors are averaged on each dimension of $\boldsymbol{R}$ to obtain the prediction for each dimension.

Yang et al. (2023a) combined KNN and ABE to predict the actual pattern of the linear arrays. Adjacent element spacings are taken as features, and AEP is taken as labels predicted by Eqs. (18) and (19).

$$\hat{g_n}(\theta) = \frac{1}{K_n} \sum_{\bar{d}_i \in \mathbb{N}} g_j(\theta) \tag{18}$$

$$\hat{g_n}(\theta) = \sum_{\bar{d}_i \in \mathbb{N}} \frac{l_i^{-1}}{\sum_{i=1}^{K_n} (l_i^{-1})} g_j(\theta) \tag{19}$$

where θ is radiation angle, $\mathbb{N}$ is neighbor set with size of K_n, l_i is distance to the neighbor, $\bar{d}_i$ is features and g_n is AEP of nth element. They further applied this model to synthesis array with varied element numbers.

4.2.5. B-spline based modeling

Sample $\boldsymbol{R}$ uniformly with reduced size, build ML models with sampled response and interpolate predictions with B-spline.

Sharma et al. (2022) combined B-spline and GPR to model gain performance in the principal plane of a monopole antenna. Dielectric constant values are taken as features. They further applied this model to synthesis single-beam and multiple-beam patterns.

4.3. Applications

To apply these response modeling methods to assist optimization (data-driven EAs), they have their own advantages and limitations. For linear array beampattern synthesis, Cui et al. (2021) applied ANN to predict far-field pattern and optimized design pattern to be consistent with the desired pattern. Real-time array synthesis can be potentially achieved since model was pretrained offline. But it suffers that a large number of training samples are required to promise model quality since ANN has 3602 output nodes with masses of model parameters. To optimize S_{11}, gain and bandwidth of Fabry–Perot resonator antenna in a multiobjective manner, Xiao et al. (2018) applied TF to parameterize three response and used ANN to learn mapping from design parameters to TF coefficients. A small number of samples are able to make good predictions since size of TF coefficients is much smaller than response and ANN has low complexity. But it suffers that it is hard to prepare dataset for training. Wu et al. (2021a) applied GPR to predict pattern for array synthesis. Dataset with vectorized label was first transformed into scalar label by adding angle feature. It makes it possible to optimize pattern with much fewer initial samples, but the size of the converted dataset is too large, which brings challenges to GPR training.

To have a more intuitive view of engineering application, we investigate typical research of response modeling in Table 5 and make a comprehensive comparison in terms of model and application in optimization. The Model enumerates dataset size, ML methods used, the features and labels of antenna instance. Application enumerates antenna type and antenna optimization based on corresponding response model, where online indicates that new data is simulated and model is updated, offline indicates that optimization is only performed on response model.

4.4. Remarks

Based on above discussion, we comment on highlights and limitations of different response modeling methods in Table 6.

5. Machine learning assisted antenna optimization with specification modeling

5.1. Motivation

ML methods are able to overcome expensive cost and nonlinear difficulties and widely used to model antenna specification, which normally predicts objectives and constraints of antenna optimization problems before full-wave simulation. Specification modeling mainly accelerates and guides optimization, which has been deeply integrated with EC and other mathematical optimizers. To outline ML-assisted optimization and distinguish from antenna response modeling, specification modeling for antenna optimization is reviewed according to ML method used.

5.2. Methods

5.2.1. SVR assisted specification modeling for antenna optimization

Ayestaran and Las-Heras (2005) applied SVR to array synthesis, where relationship between array feed values and the radiated field is learned. Zheng et al. (2011) applied SVR to design rectangular patch antenna, where SVR is used to predict resonant frequency, gain and VSWR. Roy et al. (2017) applied SVR to design slotted microstrip antennas, where resonant frequency, gain, directivity and radiation efficiency is predicted. SVR has also been applied to model reflectarray antennas (Prado et al., 2018a,b, 2023).

5.2.2. GPR assisted specification modeling for antenna optimization

Liu et al. (2014) proposed surrogate model assisted differential evolution for antenna synthesis (SADEA), where GPR is combined with

Table 5
Details of various response modeling methods.

References	Method	Model				Application in optimization	
		Dataset size	ML	Features	Label	Antenna type	Optimization mode
Chen et al. (2017)	Direct modeling	60	Kriging	Geometry	S_{11}	E-shaped patch antenna	Online
Cui et al. (2021)	Direct modeling	5000	ANN	Array excitations	Far-field pattern	Ideal or actual array	Offline
Gong et al. (2020)	Direct modeling	600	MLP	Array distribution	AEP/S parameters	Patch array	Offline
Jiao et al. (2023)	Direct modeling	550	MLP	Geometry, array distribution	Radiation pattern/S parameters	mmWave array	Online
Xiao et al. (2018)	Parametric modeling (TF)	64	ANN	Geometry	S_{11}/Gain/Radiation pattern	Fabry–Perot resonator antenna	Offline
Xiao et al. (2019)	Parametric modeling (TF)	49	ANN	Geometry	S_{11}/Pattern	Finite periodic arrays	Offline
Chen et al. (2020)	Parametric modeling (TF)	286	ANN	Array distribution	AEP	Five-element unequally spaced array	–
Kim et al. (2007)	Parametric modeling (GF)	135	MLP	Geometry	Input resistance	Loop-based broadband antenna	Online
Wu et al. (2023)	Parametric modeling (DFT)	17	Kriging	Geometry	S_{11}/Gain	Bandwidth-enhanced patch antenna	Online
Angiulli et al. (2007)	Additional feature modeling	4000	SVR	Geometry, frequency	Resonant input impedance	Printed microstrip antenna	–
Wu et al. (2021a)	Additional feature modeling	28 960	GPR	Geometry, frequency, angle	AEP	Various array	Online
Jacobs and Koziel (2013)	Additional feature modeling	91 × 3	GPR	Geometry, frequency	S_{11}	Slot dipole antenna	Online
Wu et al. (2020b)	Additional feature modeling	15 × 41	GPR	Geometry, frequency, angle	S_{11}/Gain	Single-band microstrip antenna	Online
Yang et al. (2023a)	KNN based modeling	300 × 17 × 2	KNN	Array distribution	AEP	17-element array	Online
Sharma et al. (2022)	B-spline based modeling	1050	GPR	Dielectric constant values	Gain pattern	Dielectrics around monopole antenna	Offline

Online: new data is sampled and model is updated. Offline: No data is sampled.
–: Not available.

Table 6
Comments on the highlights and limitations of different response modeling methods.

Methods	Highlights	Limitations
Direct modeling	(1) Direct modeling is very intuitive and easy to implement with ML methods. (2) Little knowledge of antenna design is required.	(1) For single-output model, quite a few models are built to predict each dimension of $\boldsymbol{R}$ so that model cost is excessively high and model errors can be accumulated. (2) For multiple-output model, model complexity becomes excessively high and masses of data are required, which increases the cost of data acquisition and model training.
Parametric modeling	(1) Since transformed parameters have shorter vector length, good enough generalization performance can be obtained by training with a small amount of data. (2) Predicted response is less insensitive to overfitting or underfitting since predictions are made on transformed parameters.	(1) It is hard to predefine order of TF, basis number of GF and parameters of DFT. Considerable prior knowledge or antenna knowledge is required. (2) Order-changing problem is further involved that the length of transformed parameter vectors for each sample is not same, which brings challenges to ML methods.
Additional feature modeling	(1) Physical meanings of response vector index are fully exploited and the difficulty of "curse of dimensionality" is overcome. (2) It can provide sufficient data for ML methods based on a small number of samples.	(1) The converted data set is overabundant that model training is extremely expensive, especially GPR. (2) The converted data set has single diversity in original features and redundant in additional features.
K-nearest neighbor based modeling	(1) KNN has high computational efficiency with limited data. (2) Little knowledge of antenna design is required.	(1) Relatively large data set is required to obtain reliable predictions. (2) It is sensitive to the method of distance calculation and number of neighbors.
B-spline based modeling	(1) Model cost is directly reduced with sampled response. (2) Little knowledge of antenna design is required.	(1) There is much loss of information on raw data. (2) It is hard to deal with highly nonlinear problems.

LCB to precreen DE population. A series of SADEA have been developed later. Radial basis function (RBF) with trust-region method for local search is integrated into SADEA (Liu and Koziel, 2015). A self-adaptive GPR modeling method with RBF local optimization was introduced to reduce training cost (Liu et al., 2021). SADEA was extended to multifidelity antenna optimization (Liu et al., 2017) and parallel optimization (Akinsolu et al., 2019). Self-adaptive Bayesian neural network is introduced to replace GPR (Liu et al., 2022).

Jacobs (2014) applied GPR to model the resonant frequency of dual-band microstrip antenna. Wang et al. (2021) applied GPR and GA to optimize dual-polarized base station antenna. Chen et al. (2022) proposed a multibranch ML assisted optimization method that GPR is combined

Table 7
Summary of ML-assisted specification modeling for antenna optimization.

References	Antenna type	ML algorithm	Optimization algorithm
Zheng et al. (2011)	Rectangular patch antenna	SVR	–
Liu et al. (2014)	Inter-chip wireless antenna, two-dimensional array	GPR	SADEA
Liu and Koziel (2015)	Microstrip array	GPR, RBF	SMAS-L
Liu et al. (2021)	Base station antenna	GPR	TR-SADEA
Liu et al. (2017)	Yagi–Uda antenna, linear microstrip antenna array	GPR	SADEA-II
Akinsolu et al. (2019)	Hybrid dielectric resonator antenna	GPR	PSADEA
Wang et al. (2021)	Dual-polarized base station antenna	GPR	GA
Zhou et al. (2020)	Asymmetric coplanar strip-fed monopole antenna	GPR	Trust-region method
Zhang et al. (2020)	Dipole antenna, cavity-backed slot antenna	GPR	SA-QNEGO
Dong et al. (2017)	Planar multiband antenna	Kriging	M-MOEA/D
Budak et al. (2021)	Analog circuit	ANN	DE
Tak et al. (2018)	W-band slotted waveguide array	MLP	–
Papathanasopoulos et al. (2023)	Yagi-Uda antenna, dual-band slotted patch antenna	ANN	ONN
Fu et al. (2022)	Slot antenna, linear antenna array	Kriging, RBF	SAPSO-mixP

with multiple constants LCB for parallel antenna optimization. Zhou et al. (2020) combined GPR and trust-region method to parallel antenna optimization. Zhang et al. (2020) proposed a surrogate-assisted quasi-Newton enhanced global optimization algorithm, where GPR is trained with low fidelity data and optimized by Quasi-Newton enhanced DE. Koziel et al. (2014) built kriging model with low-fidelity data and co-kriging model with multi-fidelity data to assist multiobjective design of ultrawideband antenna. Dong et al. (2017) proposed to combine kriging model with MOEA/D to optimize S_{11} and footprint of planar miniaturized multiband antenna.

5.2.3. ANN assisted specification modeling for antenna optimization

Budak et al. (2021) integrated ANN into ML-assisted optimization framework with low training cost and high accuracy. Tak et al. (2018) applied ANN to optimize W-band slotted waveguide array antenna, where MLP predicts the sum of the S_{11}, SLL and backlobe level. Papathanasopoulos et al. (2023) proposed an ANN-assisted optimization algorithm, where ANN is iteratively trained online to select new designs for simulation. Peng and Chen (2024b) combined a sparse ANN and an improved quantum GA to optimize antenna, which improves the algorithm's search ability with a small population size.

5.2.4. Other ML methods assisted specification modeling for antenna optimization

In addition to the above applications, some other ML methods or multi-surrogate methods have also been applied to antenna optimization. Liu et al. (2011) proposed to assist DE by GPR with expected improvement for global exploration and ANN for local exploitation. Fu et al. (2022) proposed PSO with mixed prescreening by RBF and kriging model. Zhao et al. (2021) combined RBF with self-adaptive DE to synthesize array pattern nulling. Peng and Chen (2024a) combined an enhanced diploid GA and local RBF to optimize antenna arrays, which simplifies the correlation between antenna parameters and performance, consequently decreasing the required sample size. Li et al. (2022) proposed an online data-driven enhanced-XGBoost method for antenna optimization. KNN has also been applied to online antenna optimization (Cui et al., 2020) and design of double T-shaped monopole antenna (Sharma et al., 2020).

5.3. Summary

To have a more intuitive view of engineering application, we investigate typical research of specification modeling for antenna optimization in Table 7 and make a comparison in terms of antenna type, ML algorithm and optimization algorithm.

5.4. Remarks

Machine learning-assisted evolutionary optimization has been widely developed to model and optimize antenna specification. Among ML methods, GPR is widely used because it is very suitable for the optimization with limited expensive data and able to provide uncertainty of prediction. ANN is usually for local approximation. SVR distinguishes itself in terms of model efficiency. Some other ML methods have also been incorporated for antenna design.

6. Discussion

Based on above three parts of the review, a comprehensive introduction of both ML and EC for antenna design is provided that area of circle in Fig. 6 is fully covered (blue shaded area for Section 3, red upper semicircle area for Section 4, red lower semicircle area for Section 5). Differences between response modeling and specification modeling are highlighted (Sections 4 and 5). Due to space limitation, this paper does not introduce the combination of traditional mathematical methods and EC, ML in antenna design too much.

Although ML and EC have empowered antenna design with encouraging progress and applications (Tables 3, 5, 7), there is still a long way to go for AI-assisted antenna design. This section briefly summarizes some bottlenecks and suggests promising directions, which is outlined in Fig. 12. Emerging trends are also suggested.

6.1. Challenges

Expensive cost. Fullwave solver of complex antenna is still time-consuming. Although existing response modeling methods (Table 5) have tried to replace EM simulation software based purely on ML, they usually only predict one or several responses for a certain type of antenna, which is not general and has its own limitations (Table 6) that it cannot completely replace EM simulation. Therefore, developing a general and accurate response modeling method with low cost is still an urgent challenge to be solved.

Masses of design variables. Since modern antennas become increasingly complex, the number of design variables has been growing rapidly. This poses challenges to both ML modeling and EC search that design space becomes explosively-growing. Although existing work has tried to reduce the design parameters from the perspective of parameter sensitivity analysis (Li et al., 2022), it is hard to handle the case that a large number of design parameters have significant impact, such as synthesis of large scale planar array. Therefore, modeling and optimization of antennas with a large number of design variables remains a challenge.

Numerous design specifications. Complex antenna design also puts forward higher requirements for design specifications. Existing antenna designs often stick to the requirements of a few design specifications (Table 3). Although specifications can be optimized step by step (Jiao et al., 2023) when there are more than three design specifications, numerous design specifications brings challenges of accumulated model prediction errors and the high cost of model training (Budak et al., 2021). Therefore, when the number of design specifications

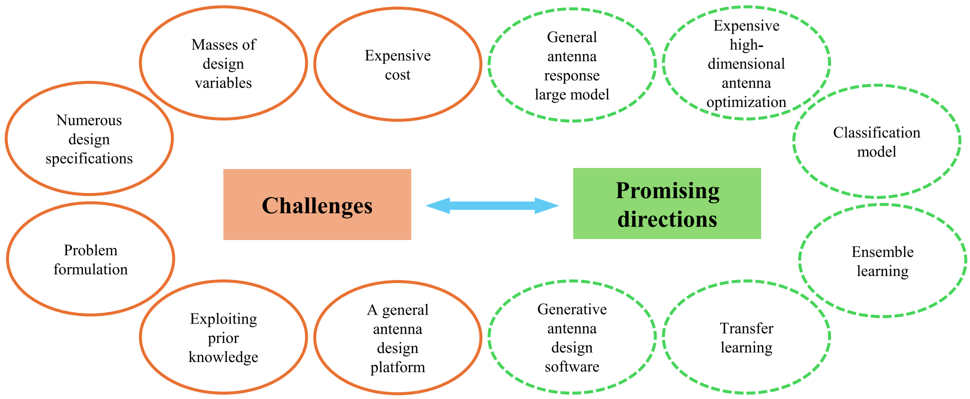


Fig. 12. Summary of challenges and promising directions.

comes to dozens or even hundreds, how to get a better and satisfactory set of designs is still a challenge.

Problem formulation. Problem formulation is even the most important and difficult challenge since beginning with wrong assumptions normally leads to worthless results and wastes a lot of time and computing resources. Existing problem formulation is often based on the experience of engineers (Table 3), which may lead to significant negative effects. Therefore, how to leverage the existing knowledge to formulate optimization problem and make it more in line with engineering problem remains a challenge.

Exploiting prior knowledge. Incorporating priori knowledge into the modeling and optimization of antenna have been proven to speed up and improve the design to a great extent (Hong et al., 2022; Wu et al., 2022a; Chen et al., 2023). The knowledge that can be used generally includes antenna domain knowledge, knowledge learned from existing data, expert experience knowledge, algorithm knowledge, etc. However, there are still questions that need to be answered, including what knowledge is worth using and how it can be used more effectively.

A general antenna design platform. A general antenna design platform is urgently needed for both industry and academia. Existing antenna design tools include EM simulation software (Fedeli et al., 2019) and antenna optimization software (Jin et al., 2018). Although there are some antenna optimization platforms that call on EM simulation software, they do not include enough antennas and optimization algorithms. Therefore, developing an open, general design platform that covers most of the existing AI antenna progress is a challenge.

6.2. Promising directions

General antenna response large model. Building a general antenna response large model is the most attractive way to replace EM simulation and overcome expensive cost difficulty. Specifically, it is necessary to first establish a general stable antenna database to store big data to provide conditions for the model, then it needs better hardware computing capabilities such as computer clusters, and finally it is necessary to develop some special AI architecture for big data. Candidate possible implementation is suggested in Appendix, including dataset preparation, model design and test, model training.

Expensive high-dimensional antenna optimization. Since it is difficult to solve the problem that the dimensions of design variables are too high from the perspective of antenna engineering practice, application of some new AI algorithms is expected to solve this difficulty. For example, encouraging results have been obtained in solving expensive high-dimensional optimization problems with thousands of design variables (Sun et al., 2017; Tian et al., 2018; Sun et al., 2022).

Classification model. Since main concern of antenna design is to obtain the optimal antenna design at the smallest cost and the specific value of antenna performance in the optimization process is not very concerned, it is possible to leverage classification model to sort different antenna designs and guide evolutionary optimization, which typically avoids modeling numerous antenna specifications. For example, classification models that have been applied to solve expensive optimization problems (Pan et al., 2018; Hao et al., 2022) are promising choice for antenna design.

Ensemble learning. Ensemble learning can provide more reliable and accurate performance than a single learner. It has been successfully applied in antenna design (Wang et al., 2022). Modeling different antenna problem formulations by heterogeneous ensemble learning is expected to reduce bias.

Transfer learning. Transfer learning can make full use of prior knowledge by taking advantage of the similarities between learning tasks. There have been attempts to use transfer learning in antenna design (Guo et al., 2020). To transfer antenna domain knowledge and avoid negative transfer is a promising choice in antenna design.

Generative antenna design software. Shi et al. (2022) proposed an antenna synthesis system that inputs desired design specifications and outputs an antenna design that meets the requirements, which provides the forerunner for general antenna software. Recently, large generative model has raised much attention due to their excellent ability to emancipate the productive forces (Radford et al., 2018). Therefore, it is a promising direction to develop a generative antenna design software that can design antenna according to user requirements automatically. For example, develop the antenna synthesis basic intelligence software (AntsynGPT) to form an interpretable and canonical antenna dynamic optimization theoretical system, which can provide the antenna design core functional sub-modules for the general AI.

6.3. Emerging trends

Recently, thanks to the development and popularity of ANN or deep learning techniques, several efforts have emerged in the antenna and propagation community to combine state-of-the-art deep learning techniques with corresponding antenna designs. For example, Yang et al. (2023c) applied deep neural networks to facilitate the circularly polarized antenna array synthesis with mutual coupling. He et al. (2023) combined ANN and simulated annealing algorithm to design the wideband patch antenna. Yang et al. (2023b) applied ANN to real-time pattern synthesis for large-scale conformal array. Zhao et al. (2023) combined ANN and inverse fast Fourier transform to realize efficient beampattern synthesis for large-scale time-modulated arrays. Sun et al. (2024) proposed an efficient iterative method assisted by ANN for pattern synthesis of arbitrary conformal arrays. Bai et al. (2024) applied improved conditional generative adversarial network to real-time diagnosis of impaired conformal antenna arrays. Convolutional Neural Network has been applied to antenna geometry design (Wu et al., 2024). Convolutional Neural Network-Long Short Term Memory network has also been applied to antenna modeling (Wei et al., 2023; Zhu et al., 2024b).

7. Conclusion

This paper presents a comprehensive overview of both ML and EC for antenna modeling and optimization. Theoretically speaking, this survey provides a complete guide for inspiring new AI antenna design methods. Practically speaking, this survey is expected to promote the application and implementation of some new method theories in the field of antenna design. This paper contributes to quickly investigate the application of ML and EC in antenna design for antenna and propagation community, and gives a guidance to build different types of EM simulation surrogate models, including response modeling and specification modeling.

Over past decades, EC has made great progress in antenna design. According to different problem formulation, various types of EAs have been successfully applied to single-objective antenna optimization, multiobjective antenna optimization and constrained antenna optimization. ML have demonstrated its advantage to accelerate antenna optimization design by building surrogate model to learn the mapping between antenna design parameters and antenna performance. In the field of antenna design, two kinds of ML surrogate models of EM simulation have been applied, including response modeling and specification modeling, which can both greatly accelerate the process of antenna design and promote automatic antenna design to intelligent antenna design.

Although AI-assisted antenna design has made great progress, there is still limitations for development. For example, modern antenna design becomes more and more complicated and makes design costs more expensive, which involves large-scale design parameters, design specifications and poses great challenges to existing methods. Moreover, existing antenna design does not make full use of expert experience and EM theory knowledge, which can be further used to improve the design performance. Furthermore, the development of a common noncommercial antenna design optimization platform will help to promote the development of academic and engineering in the field of antennas. Some opportunities are suggested in this field. For example, using classification model to learn the relation between different antenna designs is expected to avoid many specifications modeling and optimization. Moreover, using transfer learning to make full use of EM field knowledge and expert knowledge is expected to further improve antenna design efficiency.

CRediT authorship contribution statement

Hanhua Zou: Writing – original draft, Visualization, Software, Methodology, Investigation. **Sanyou Zeng:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization. **Changhe Li:** Writing – review & editing, Supervision, Funding acquisition. **Jingyu Ji:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix. Pseudocode of general antenna response large model

Algorithm 1 General antenna response large model

Dataset preparation:
1: Data collection: Collect data from initial data, new data generated during the optimization, data generated by EM knowledge and measurement data. Various antennas (such as patch antennas, Yagi antennas, array antennas, etc) are needed to generate data.
2: Data preprocessing: Include data classification, labeling to ensure the quality and format of the data.
3: Database construction: Construct response database according to antenna type.
Model design and test:
4: Model architecture selection: Choose model architecture for the task, such as Transformer architecture.
5: Parameter adjustment: Adjust the hyperparameters of the model, such as learning rate and batch size.
6: Model validation and testing: Use validation sets and test sets to evaluate the performance of the model and make necessary adjustments.
Model training:
7: Training process: The pre-processed data is entered into the model for training, which may require a lot of computational resources.
8: Training monitoring: Monitor the loss function value and other indicators in the training process to ensure the stability and effect of training.
9: Model evaluation and tuning: Evaluate the performance of the model using validation and test sets, and adjust the model parameters or architecture based on the evaluation results.

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