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Density-Enhanced Multiobjective Evolutionary Approach for Power Economic Dispatch Problems

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***Abstract*—Economic dispatching of generating units in a power system can significantly reduce the energy cost of the system. However, the economic dispatch (ED) problem is highly constrained, and often has disconnected feasible regions because of various physical features. Enhancing population diversity is critical for the evolutionary approach to fully explore and exploit the feasible regions. In this article, we propose a density-enhanced multiobjective evolutionary approach to solve ED problem. An ED problem is first transformed into a tri-objective optimization problem, and then multiobjective optimization techniques are employed to fully optimize the constraints and cost function simultaneously. The first two objectives are derived from the original ED problem, while the third one is a novel density objective constructed by niching methods to enhance population diversity. These three objectives are optimized simultaneously by a dynamic dominance relation, which can make a good balance among feasibility, diversity, and convergence. To evaluate the performance of this proposed approach, 22 benchmark problems and seven real-world ED problems with different features are tested in this article. The experimental results show that our approach performs better than or at least competitive to the state-of-the-art algorithms, especially on large-scale ED problems.**

***Index Terms*—Differential evolution (DE), dynamic constraint-handling technique, economic dispatch (ED) problem, multiobjective optimization.**

I. Introduction

The ECONOMIC dispatch (ED) problem for power generation is a constrained optimization problem that is commonly seen in real-world power systems [1]–[6]. There are two common objectives when solving an ED problem. The first one is to minimize the total generation cost for all generating units. The second one is to satisfy all the constraints that the power system contains. Traditionally, mathematical optimization methods, such as Lagrangian relaxation [7], quadratic programming [8], gradient method [9], linear programming [10], and hybrid approaches [11], [12], were adopted to solve the ED problems. However, the performance of these mathematical approaches overwhelmingly depends on the starting points which are problem-specific and difficult to determine in advance. Furthermore, since various physical features are involved in the power systems, such as valve-point effects, prohibited operating zones, ramp rate limits, and multiple fuel options, an ED problem may contain multiple characteristics, such as multimodality, discontinuity, nonconvexity, and large-scale dimensions, which makes it difficult for traditional optimization methods to solve the problem [4], [5].

Recently, the heuristic algorithms, such as genetic algorithm (GA) [13], particle swarm optimization (PSO) [4], [14], [15], differential evolution (DE) [16], artificial bee colony (ABC) [17], and artificial immune system (AIS) [18], have also been proposed to solve ED problems. These nature-inspired heuristic algorithms utilize a population of individuals and the principle of survival of the fittest to search for the promising solutions. They require little domain information of a given ED problem in the searching process and have been shown to have strong global search ability.

The ED problem is a highly constrained problem that involves a number of equality and inequality constraints. Therefore, various constraint-handling techniques [19] were proposed and developed to be integrated with heuristic algorithms for solving ED problems. According to [20], the popular constraint-handling techniques developed over the years can be generally classified into three categories as follows.

1) Methods based on penalty functions [13], [21], [22].
2) Methods based on the preference of feasible solutions over infeasible solutions [20], [23], [24].
3) Methods based on multiobjective optimization [10], [25], [26].

Due to the simplicity, the penalty function is one of the most frequently used techniques for handling constraints. It converses the constrained optimization problem into an unconstrained one by adding a punishment. In this way, the original heuristic algorithms, such as GA, DE, and PSO, can be directly integrated without any further operation. As for the second category, the comparison between two individuals is based on their feasibility. For instance, in the selection of feasibility rule [27], the feasible solutions are prior to the infeasible solutions, even if the latter ones slightly violate the constraints but have good objective function values. In the methods based on multiobjective optimization, the multiobjective optimization

Manuscript received July 18, 2019; accepted October 29, 2019. This work was supported by the National Natural Science Foundation of China under Grant 61502544 and Grant 61332002. This article was recommended by Associate Editor H. Tianfield. *(Corresponding author: Wei-Jie Yu.)*

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This article has supplementary downloadable material available at http://ieeexplore.ieee.org, provided by the author.

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Digital Object Identifier 10.1109/TSMC.2019.2953336

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techniques are applied to solving constrained optimization problems. By the concept of considering the constraints as objectives [28], [29], a multiobjective optimization problem (MOP) is constructed from the original objective function and the information of constraints. Then, the existing successful multiobjective optimization algorithms can be utilized to solve the transformed problem.

Among the above three categories, there are two distinct features [30], [31] when applying multiobjective optimization techniques to solving constrained optimization problems, especially to ED problems. First, the constraints and the objective function are optimized in a synchronous way so that some infeasible solutions with better objective values and less constraint violations can be selected. As a result, the search bias caused by handling constraints and optimizing objective function in an asynchronous way, such as the methods based on the first and second categories, can be efficiently avoided. Second, it is well known that a good population diversity can alleviate premature convergence. Nonetheless, for solving ED problems by multiobjective optimization, less effort has been made to enhance population diversity. Comparing with the other two categories, multiobjective optimization has advantage of maintaining population diversity. To make full use of multiobjective optimization, in this article, we develop a density-enhanced multiobjective optimization approach (DMOA) to obtain high-quality feasible solutions. In the proposed approach, the ED problem is first transformed into a tri-objective optimization problem (TOP), and then multiobjective optimization combined with DE is employed to solve the transformed problem. In the transformation, the niching method is utilized to enhance the population diversity. Generally, the main contributions of the proposed DMOA are summarized as follows.

1) In the proposed transformation, a novel density objective is constructed by niching methods to enhance population diversity in both infeasible and feasible regions.
2) A shrinkage scheme is proposed to alleviate the search premature convergence in the infeasible regions. The infeasible regions are conditionally considered as feasible regions, so that search bias can be relieved.
3) By transforming an ED problem into a TOP, the transformed three objectives can be optimized simultaneously. In this way, a good balance among feasibility, diversity, and convergence can be obtained by multiobjective optimization.

The remainder of this article is organized as follows. Section II reviews related works on constraint-handling techniques and population diversity maintaining strategies. Section III presents the mathematical formulations of ED problems. Preliminaries of our proposed algorithm are reviewed in Section IV. Section V introduces DMOA. Experiments and performance comparisons among several state-of-the-art algorithms are reported in Section VI. Moreover, the effectiveness of the third objective and shrinkage scheme in DMOA are also analyzed in Section VI. Section VII concludes this article.

II. Literature Review

In recent decades, how to properly utilize different kinds of energy has attracted research efforts due to the rapidly increasing consumption. Therefore, the ED problem in power systems has become an important research issue [2], [15]. Since the power ED problem is highly constrained, handling constraints is critical in solving ED problems. In addition, the objective function is also important. When generating certain amount of power, the lower objective function value the algorithm obtains, the less energy consumption it brings. Comparing to the classic methods [7]–[10], the nature-inspired heuristic algorithms have shown their advantages of handling constraints and optimizing objective function in solving ED problems. In this section, we mainly focus on how these heuristic algorithms handle constraints and maintain population diversity for global optimum.

As introduced in Section I, the penalty function is the most widely used technique. The penalty function and its variants mainly incorporate the constraint violation as a punishment to objective function value. Therefore, the constraints and objective function information are combined to form a new fitness function. A common formulation is written in the following form [22]:

$$\text{Minimize}\quad \psi(\mathbf{x}) = f(\mathbf{x}) + \omega_0 \sum_{j=1}^{m} \omega_j \big(G_j(\mathbf{x})\big)^{\beta} \tag{1}$$

where $\psi(\mathbf{x})$ is the fitness value of $\mathbf{x}$, ω_0 and ω_j are penalty factors, and $G_j(\mathbf{x})$ [24] is the constraint violation on the jth constraint

$$G_j(\mathbf{x}) = \begin{cases} \max\{g_j(\mathbf{x}), 0\}, & \text{for inequality constraint} \\ \max\{|g_j(\mathbf{x})| - \delta, 0\}, & \text{for equality constraint} \end{cases} \tag{2}$$

where $G_j(\mathbf{x})$ is the degree of constraint violation of the variable vector $\mathbf{x}$ on the jth constraint g_j, and δ is a positive tolerance parameter for the equality constraints.

In [21], [32], and [33], the PSO, and the random drift PSO (RDPSO) and its improved version ST-IRDPSO are integrated with the penalty function method to solve ED problems. To avoid penalty function being ill conditioned in some specific situations, Chiang [13] developed a new penalty function method by employing the Lagrange function. Based on the penalty function method, existing heuristic algorithms usually enter the feasible region first, and then locate the feasible solution with the minimum objective value. In the early phase, most solutions violate the constraints, which means the constraint violation plays a dominant role in the fitness function. Hence, the population always moves toward a region in which all constraint violations are small. Once the population enters the feasible region, the objective function value plays a dominant role in the fitness function. Then the solution with the minimum objective value would be located in the later phase.

In addition, constraint-handling methods based on the preference of feasible solutions over infeasible solutions is another popular way for solving ED problems. In general, feasible solutions are mainly considered to be superior to infeasible ones such as the feasibility rule proposed in [27]. However, due to the physical features existed in ED problems, there are too many mutually coupled equality constraints that make the naive feasible rule fail to work efficiently. Therefore, various new variants were specially developed for ED problems. For example, Duvvuru and Swarup [16] utilized only the feasible solutions to find the feasible region first, and then DE was employed to locate the optima in the certain region. Park *et al.* [34] proposed an adaptive mechanism to treat the constraints. Two tolerance thresholds for equality and inequality constraints are set. The infeasible solutions are modified iteratively till their constraints are less than the certain thresholds. Zaman *et al.* [15] utilized the ε-constrained method to address the selection pressure from equality constraints. The tolerance for equality constraints is controlled by the ε-parameter. In initial phase, the tolerance is set to the maximum constraint violation according to the initial population, and then the tolerance gradually shrinks into zero during the evolutionary progress. In these ways, the search is mainly oriented toward the direction of the minimum constraint violation, meanwhile, some useful information provided by objective function is adopted to alleviate the greediness.

Recently, multiobjective optimization [35]–[37] has also been adopted to solving ED problems. The following part describes one of the implementations:

$$\text{Minimize} \ \ \mathbf{F}(\mathbf{x}) = (F_T, G(\mathbf{x})) \tag{3}$$

where $G(\mathbf{x})$ is the overall constraint violation on the variable vector $\mathbf{x}$ and can be calculated as follows:

$$G(\mathbf{x}) = \sum_{j=1}^{m} \frac{G_j(\mathbf{x})}{G_{\max,j}} \tag{4}$$

where m is the total number of equality and inequality constraints, and $G_{\max,j}$ is the maximum violation on the jth constraint obtained in the initialization phase.

In this implementation, all the constraints are treated as a separated objective [29]. In this way, an ED problem is converted into an MOP.

There is an dominance relation defined between any two individuals in multiobjective optimization. Given two variable vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, if $\mathbf{f}(\mathbf{x}_1) \leq \mathbf{f}(\mathbf{x}_2)$ with at least one strict inequality, then $\mathbf{x}_1$ is said to dominate $\mathbf{x}_2$. Moreover, if there does not exist any other variable vector $\mathbf{x}_1 \in \Omega$ such that $\mathbf{x}_1$ dominates $\mathbf{x}_2$, then $\mathbf{x}_2$ is called a nondominated solution. All nondominated solutions form the Pareto set (PS). The Pareto front (PF) is a set which is defined as $\{\mathbf{f}(\mathbf{x})|\mathbf{x} \in \text{PS}\}$ [38].

The schematic of solving ED problems by multiobjective optimization is plotted in Fig. 1. The point A in Fig. 1 is the optimal solution of an ED problem, since its overall constraint violation is 0, and its objective value is minimum. Meanwhile, the point A is also an endpoint of the PF. When the PF of (3) is found, the optimal solution A of an ED problem is also found.

In [26], [28], and [29], the multiobjective optimization as a constraint-handling technique was proposed to solve single objective constrained optimization problems. The constraints are treated as either independent objectives or an integrated objective. Due to a good balance between population diversity and convergence, the multiobjective optimization evolutionary algorithm (MOEA) can avoid premature convergence and local optima efficiently. Combing with GA [39], PSO [40], [41], and DE [42], [43], various variants based on multiobjective optimization have been developed to solve ED problems. Furthermore, in [15], a nondominated and crowding distance mechanism was proposed as a selection strategy to deal with objective function and constraints simultaneously. In [44], multiobjective optimization is combined with feasibility rules to handle constraints.

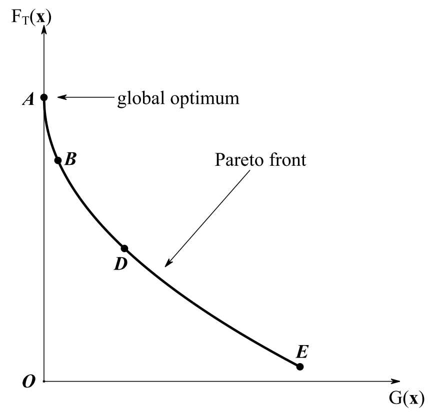

Fig. 1. Schematic of solving ED problems by multiobjective optimization. The arc AE is the PF of (3). The global solution (A) of the original ED problem lies on the endpoint of the PF.

Moreover, for money savings, the global optimum is always desirable in solving ED problems. To avoid local optima, how to improve population diversity is another issue in ED problems. So far, many efforts have been made. Ciornei and Kyriakides [45] adopted an information sharing strategy to keep population diversity in the search space. Vlachogiannis and Lee [46] proposed a new concept of coordinated aggregation to distribute the swarms. Selvakumar and Thanushkodi [47] adopted the local random search to exploit the promising area. In this way, the promising area identified in the early stage cannot be missed. Sun *et al.* [33] preserved population diversity by simulating the behavior of an electron moving in a metal conductor placed in an external electric field. Aragón *et al.* [23] used two versions of redistribution power operators to keep population diversity in the feasible regions that the search has found. As a result, the proposed approach can provide high-quality optimal solutions for ED problems.

For more details of constraint-handling techniques and population diversity maintaining strategies for ED problems, readers are referred to [4] and [5].

III. Formulation of the ED Problem

A. Objective Function of the ED Problem

The classic objective function of the ED problem is the total fuel cost of all generating units [21]. Without loss of generality, the mathematical formulation of the classic objective function

[13], [14], [32], [33] can be expressed as follows:

$$\text{minimize} \quad F_T = \sum_{i=1}^{N} F_i(P_i) \tag{5a}$$

$$\sum_{i=1}^{N} F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \tag{5b}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{5c}$$

where F_T is the total generation cost of a power system, i is the index of generating units, N is the number of generators, F_i, P_i, $P_i^{\min}$, and $P_i^{\max}$ are the cost function, power output, minimum and maximum power outputs of the generation i, respectively, and a_i, b_i, and c_i are the cost coefficients.

1) ED Problem With Valve-Point Effects: The valve-point effects in large generating units make the fuel cost curve nonlinear. To simulate the valve-point effects in the mathematical model, the sinusoidal function is employed. Therefore, the quadratic function (5b) is modified as follows [13]:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 + \left| e_i \times \sin\left(f_i \times \left(P_i^{\min} - P_i \right) \right) \right| \tag{6}$$

where e_i and f_i are the cost coefficients of valve-point effects on generator i. It is obvious that if the valve-point effects are considered in an ED problem, the objective function will become nonconvex due to the sinusoidal function in (6).

2) ED Problem With Multifuels: In the real-world power systems, the dispatching units are usually operated with multifuel sources. Hence, each unit should be represented with several piecewise quadratic functions to reflect the effects of different fuel types. The fuel cost function in the piecewise formulation can be practically described as follows [13]:

$$F_i(P_i) = \begin{cases} a_{i1} + b_{i1} P_i + c_{i1} P_i^2, \ \text{fuel } 1, & P_{i1}^{\min} \le P_i \le P_{i1}^{\max} \\ a_{i2} + b_{i2} P_i + c_{i2} P_i^2, \ \text{fuel } 2, & P_{i2}^{\min} \le P_i \le P_{i2}^{\max} \\ \vdots & \vdots \\ a_{ij} + b_{ij} P_i + c_{ij} P_i^2, \ \text{fuel } j, & P_{ij}^{\min} \le P_i \le P_{ij}^{\max} \end{cases} \tag{7}$$

where j is the index of fuel types, a_{ij}, b_{ij}, and c_{ij} are the cost coefficients of the generator i for the jth fuel type, $P_{ij}^{\min}$ and $P_{ij}^{\max}$ are the minimum and maximum power outputs of generator i by using the jth fuel type, and specifically, $P_{i1}^{\min} = P_i^{\min}$, $P_{ij}^{\max} = P_i^{\max}$, and $P_{ik-1}^{\max} = P_{ik}^{\min}$ for $k = 2, \ldots, j$.

3) ED Problem With Valve-Point Effects and Multiple Fuels: To obtain an accurate and practical solution for ED problems, both valve-point effects and multifuels should be considered in the mathematical formulation simultaneously. In this article, the cost function integrated with (6) and (7) can be realistically formulated in the following form [13], [21]:

$$F_i(P_i) = \begin{cases} F_{i1}(P_i), & \text{fuel } 1, \ P_{i1}^{\min} \le P_i \le P_{i1}^{\max} \\ F_{i2}(P_i), & \text{fuel } 2, \ P_{i2}^{\min} \le P_i \le P_{i2}^{\max} \\ \vdots & \vdots \\ F_{ij}(P_i), & \text{fuel } j, \ P_{ij}^{\min} \le P_i \le P_{ij}^{\max} \end{cases} \tag{8}$$

where

$$F_{ij}(P_i) = a_{ij} + b_{ij} P_i + c_{ij} P_i^2 + \left| e_{ij} \times \sin\left(f_{ij} \times \left(P_{ij}^{\min} - P_i \right) \right) \right| \tag{9}$$

and e_{ij} and f_{ij} are cost the coefficients of valve-point effects on the generator i for the jth fuel type.

B. Constraints of the ED Problem

In this article, the following constraints are considered to formulate the final problem.

1) Active Power Balance Equation: To make power balanced, the total power generated by the system should be equal to the load demand of the system and the transmission losses [8], [9], i.e.,

$$\sum_{i=1}^{N} P_i = P_D + P_L \tag{10}$$

where P_D is the total system load demand, and P_L is the total transmission network loss. P_L is usually approximated by the Kron's loss function [33], which is stated as follows:

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_i + \sum_{i=1}^{N} B_{0i} P_i + B_{00} \tag{11}$$

where B_{ij}, B_{0i}, and B_{00} are the loss coefficients or B-coefficients.

2) Ramp Rate Limits: In the operating process of each online unit, its operating range is restricted by ramp rate limits. According to [33], the ramp limits can be expressed by the inequality constraints as follows:

$$P_i - P_i^0 \le UR_i \text{ and } P_i^0 - P_i \le DR_i \tag{12}$$

where P_i^0 is the previous power output of the generator i, and UR_i and DR_i are the up-ramp and down-ramp limits of the generator i, respectively.

If the power output limits (5c) and the ramp limits (12) are taken into account simultaneously, these two inequality constraints can be rewritten as follows:

$$\max\left\{ P_i^{\min}, \ P_i^0 - DR_i \right\} \le P_i \le \min\left\{ P_i^{\max}, \ P_i^0 + UR_i \right\}. \tag{13}$$

3) Prohibited Operating Zones: In some power systems, the whole operating range of a generator is not always available because of the physical limitations. Hence, a generator contains some prohibited operating zones in its operating range. Considering the prohibited operating zones, the operating range of generator i can be approximated by the following constraints [33], [34]:

$$P_i \in \begin{cases} P_i^{\min} \le P_i \le P_{i,1}^{l} \\ \vdots \\ P_{i,k-1}^{u} \le P_i \le P_{i,k}^{l}, \ k = 2, \ldots, N_{pz} \\ \vdots \\ P_{i,N_{pz}}^{u} \le P_i \le P_i^{\max} \end{cases} \tag{14}$$

where $P_{i,k}^{l}$ and $P_{i,k}^{u}$ are the lower and upper bounds of the kth prohibited zone of generator i, and N_{pz} is the number of prohibited operating zones that generator i contains.

IV. Preliminary Knowledge

In this article, DE and niching methods are utilized in our proposed approach DMOA. To have a better understanding, we introduce them in this section briefly.

A. Differential Evolution

As a search engine, DE [48], [49] has been widely used in many evolutionary algorithms to generate offspring. The mutation and crossover operators in DE are briefly described in this section.

1) Mutation: The most frequently used mutation strategy is DE/rand/1. Three vectors are randomly selected from the current population. Then, the mutation operator is performed on them to generate a mutant vector $\mathbf{v}$

$$\mathbf{v}_i = \mathbf{x}_{r_1} + F \cdot \left(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}\right) \tag{15}$$

where i is the sequence number, F is the scale factor, and r_1–r_3 are three integers which are selected randomly. In addition, $r_1 \neq r_2 \neq r_3 \neq i$.

2) Crossover: The crossover operator is performed to produce the trial vector $\mathbf{u}_i$ consisting of parts of the target vector $\mathbf{x}_i$ and the mutant vector $\mathbf{v}_i$. Equation (15) shows a popular binomial crossover operator

$$u_{i,j} = \begin{cases} v_{i,j}, & \text{if } \text{rand}_j(0,1) \leq C_r, \text{ or } j == j_{\text{rand}} \\ x_{i,j}, & \text{otherwise} \end{cases} \quad j = 1, \ldots, d \tag{16}$$

where C_r is the crossover rate and j_{rand} is an integer which is randomly chosen in the range $\{1, \ldots, d\}$. The condition $j = j_{\text{rand}}$ ensures the crossover operator be applied in at least one dimension.

B. Niching Method

The niching method is commonly adopted to solve problems that contain multimodal domains and require identifying multiple optimal solutions [50]. Among various niching methods, fitness sharing is a popular one [51]. As shown in Fig. 2, the red points are three optima, including one global optimum and two local optima. By punishing the individuals that inhabit densely populated regions, the population will avoid converging to a single solution, i.e., the black points are individuals scattered over the three attraction basins. In this article, the niche count m_i is utilized to construct an additional objective. The m_i can be calculated as follows:

$$m_i = \sum_{j=1}^{N} sh(d(i,j)) \tag{17}$$

where N_p denotes the population size, $d(i,j)$ is the distance between two individuals i and j, and sh is a sharing function which reflects the degree of similarity between two individuals. The most widely used form of the sh [51] is presented as follows:

$$sh(d(i,j)) = \begin{cases} 1 - (d(i,j)/\sigma), & \text{if } d(i,j) < \sigma \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where σ denotes a threshold of dissimilarity (also the niche radius).

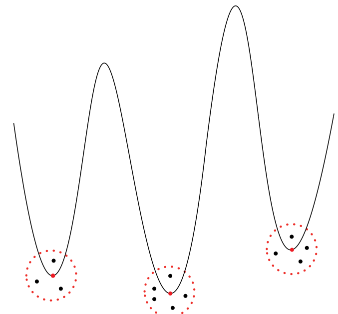

Fig. 2. Illustration of the niching method. The red dotted circles are three niches. In these niches, similar individuals will be punished in order to maintain the population diversity. The three red points are the lowest points in their respective attraction basins.

However, a critical limitation in fitness sharing is that it requires *a priori* knowledge to set the dissimilarity threshold σ.

V. Proposed Algorithm

In this section, the motivation behind DMOA is first introduced. Then, following the motivation, an ED problem is transformed into a TOP to be optimized. Finally, the dynamic multiobjective optimization technique is systematically described to solve the converted problem.

A. Motivation

ED problems often have disconnected feasible regions because of the discontinuous prohibited zones and ramp rate limits. Therefore, the optimization problem would contain a lot of local optima. Without good population diversity, the local optimum information may cause premature convergence. Two goals behind the motivation of our proposed approach are to: 1) enter the feasible regions as many as possible and 2) maintain the population diversity in each feasible region for locating the feasible optimal solution.

To better understand our motivation, we construct an ED problem with two disconnected crescent shaped (nonconvex) feasible regions in Fig. 3. To solve this ED problem, the search should find the feasible optimal solution (the point A) which has the minimum distance to the origin O. As for the two disconnected feasible regions, it can be seen that all the feasible solutions in the left crescent shaped feasible region have larger distances to O than the point A in the right crescent shaped feasible region. However, the search is easy to enter the left one which has a larger size. Therefore, the search would fall into local optima (e.g., point A') easily. Usually, an algorithm cannot distinguish these two feasible regions without *a priori* knowledge. Thus, the optimization performance becomes worse when there are multiple disconnected feasible regions.

For the first goal, we conditionally extend the feasible regions to a larger area. As shown in Fig. 3, the larger light shadow area enclosed by the arc PMQ and PNQ covers the two feasible regions. Due to the larger area, the search has access to enter both left and right feasible regions. For the second goal, population diversity needs to be maintained to locate the

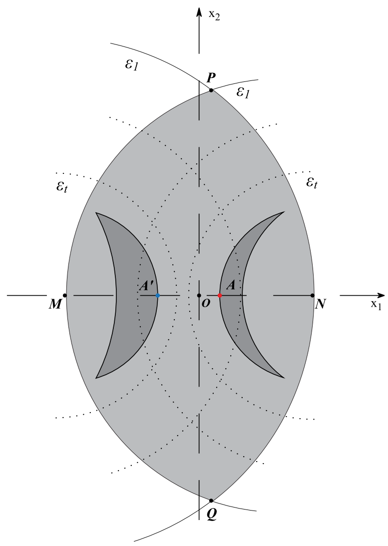

Fig. 3. Plot of two feasible regions in relation to objective values. The two crescent shaped areas are two disconnected feasible regions and the dashed concentric arcs $\varepsilon_1, \ldots, \varepsilon_k$ form an equi-infeasible contour. Points A and A' are the nearest points to the origin O in the two feasible regions, respectively.

optimal solution A and avoid local stagnation at A'. However, without *a priori* knowledge, we do not know which feasible region the population diversity should be maintained. Hence, we employ niching methods to enhance the whole population diversity, so that more individuals have the probabilities to be maintained in both left and right feasible regions. As a result, with population diversity enhanced, the potential solutions and their corresponding domains can be fully exploited.

To achieve above two goals, multiobjective optimization as a constraint-handling technique is utilized in this article. Comparing with the penalty function and the preference of feasible solutions over infeasible solutions, the multiobjective optimization handles constraints and optimizes objective function simultaneously. In this way, some infeasible solutions with better objective values and less constraint violations can be selected, and therefore, the premature convergence can be alleviated without any extra mechanism. More importantly, once population diversity has been enhanced, it can be maintained in a high level, since the multiobjective optimization already has excellent performance in balancing population diversity and convergence.

B. Tri-Objective Optimization Model

To apply multiobjective optimization, an MOP problem needs to be constructed first.

To improve population diversity, niching methods are widely employed. Usually, the niche count m_i is used as a punishment imposed on the raw objective function value. Hence, similar to the disadvantage of penalty function methods, the performance of niching methods is sensitive to the niche radius σ. On the other hand, the niching method cannot be employed on the objective function values directly when multiobjective optimization is applied, since the PF would be changed.

Moreover, if the valve-point effects are considered, the problem may contain many local optima. To locate the global optimum efficiently, the niching method is utilized to distinguish different individuals by their degrees of crowding. In this article, an additional objective named density objective is constructed based on the niche count m_i in (17). Thus, an ED problem is finally transformed into a TOP

$$\begin{aligned} &\text{minimize} \;\; \mathbf{F}(\mathbf{x}) = (F_T, G(\mathbf{x}), D(\mathbf{x})) \\ &\text{subject to} \;\; P_i^{\min} \le x_i \le P_i^{\max}, i = 1, \ldots, N \end{aligned} \tag{19}$$

where $D(\mathbf{x})$ is the proposed density objective, and $D(\mathbf{x}) = m_i$.

Afterward, in order to balance population diversity and convergence, multiobjective optimization techniques are employed to optimize these three objective functions simultaneously. When solutions are nondominated in the case of two objectives, F_T and $G(\mathbf{x})$, they are still nondominated in the case of three objectives, F_T, $G(\mathbf{x})$, and $D(\mathbf{x})$, so introducing $D(\mathbf{x})$ does not degrade the search capability. Conversely, the individuals with lower $D(\mathbf{x})$ values can survive longer, whereby the domains occupied by these individuals will be fully exploited. This is the reason why the density objective $D(\mathbf{x})$ can improve the search ability during the evolutionary process.

C. Shrinkage Scheme

However, if we remain the light shadow area unchanged, infeasible solutions in this area would survive for a long time and waste computational resources unnecessarily. Hence, we gradually shrink the light shadow area from a large size to the original feasible regions, i.e., from ε_1 to ε_k. As shown in Fig. 1, except for endpoint A, the PF of (3) contains all the nondominated solutions which violate the constraints. These solutions are undesirable in the final result. One reason is that these solutions are useless for the original ED problem, because they do not satisfy the constraints. The second reason is that removing these inferior solutions can save a lot of computational resources which would be very precious, especially when the simulation experiments are time-consuming. However, during the evolutionary process, some special inferior solutions may still carry some useful information. For example, in Fig. 1, the nondominated point B violates the constraints slightly, while its objective value is smaller than A. Its objective function information is useful to find minimum generating cost and prevent the search from falling into local optima.

To deal with the above issue, a shrinkage scheme [52] is utilized to narrow down the range of the PF. The shrinkage scheme can be modeled as follows:

$$\varepsilon_t = T_0\left(1 - \frac{t-1}{t_{\max}-1}\right)^{cp}, \;\; t = 1, \ldots, t_{\max} \tag{20}$$

where ε_t is the tolerance for the overall constraint violation in the t-th generation, T_o is an initial tolerance in the first generation, $t_{\max}$ denotes the maximum number of generations, and cp is the control parameter of the shrinkage speed.

On the other hand, the niching radius σ should also be controlled by a shrinkage scheme. There are two reasons for dynamically adjusting the niche radius σ.

1) For DMOA solving ED problems, the evolutionary process, which starts from the initial search region to the

feasible region, determines that the niche radius cannot be a constant in order to match the boundary shortening. Furthermore, when the search enters the feasible region, the final aim is to find the best feasible solution rather than a set of nondominated solutions. This determines that $D(\mathbf{x})$ should be 0 in the final generation. Hence, niche radius σ should be set to 0 to control $D(\mathbf{x})$ to be 0. Based on these factors, the niche radius should gradually shrink from an initial value to 0 in our proposed method.

2) In DMOA, the niching method is integrated with multiobjective optimization by constructing an additional objective, rather than acting on the raw objective function value as a punishment. Nevertheless, no matter which form is adopted, the niching method aims at enhancing the population diversity and deal with multimodal domains. However, without a prior knowledge, we do not know whether the valve-point effects are considered or not in a given ED problem. Hence, if the population diversity is enhanced blindly by the niching method, it even has a negative influence on the ED problem without the valve-point effects. To make the algorithm more robust, the niche radius σ is also controlled by the shrinkage scheme so that a specific niche radius cannot make a long-term impact.

Generally, the main idea of the shrinkage scheme is to set an initial boundary (tolerance) for the search region at the beginning, and then gradually shorten the boundary (tolerance) to the feasible region. In the meantime, the range of PF becomes narrower and narrower from $AE \rightarrow AD \rightarrow AB \rightarrow A$. During the shrinkage, the solutions inside the search region enclosed by the current boundary are kept evolving. This makes the multiobjective optimization only work inside the boundary. Otherwise, the solutions outside the boundary will be removed to help all individuals move toward the feasible region.

D. Dynamic Pareto Dominance Relation

The static Pareto dominance relation introduced in Section II is unsuitable to the shrinkage scheme, and hence a modified variant named dynamic Pareto dominance relation is proposed to compare two individuals $\mathbf{v}$ and $\mathbf{u}$ as follows.

1) If both $G(\mathbf{v}) \leq \varepsilon_t$ and $G(\mathbf{u}) \leq \varepsilon_t$, then the original Pareto dominance relation is used to compare $\mathbf{v}$ and $\mathbf{u}$ according to $(F_T(\mathbf{v}), G(\mathbf{v}), D(\mathbf{x}))$ and $(F_T(\mathbf{u}), G(\mathbf{u}), D(\mathbf{x}))$.
2) If $G(\mathbf{v}) \leq \varepsilon_t$ and $G(\mathbf{u}) > \varepsilon_t$, then $\mathbf{v}$ dominates $\mathbf{u}$, and vice versa.
3) If both $G(\mathbf{v}) > \varepsilon_t$ and $G(\mathbf{u}) > \varepsilon_t$, then the one with smaller overall constraint violation dominates the other.

It can be seen that the dynamic Pareto dominance relation is analogous to the original version. As for the first rule, if the overall constraint violations of the two individuals are less than the current tolerance ε_t, the two individuals are considered in a pure multiobjective optimization environment. Otherwise, the second and third rules would be preferred to push the search toward the feasible region based on the overall constraint violation.

Algorithm 1: DMOA

Initialization:
- Randomly generate an initial population with N_p individuals from decision space S.

while $t < t_{\max}$ **do**
- $t = t + 1$;
- **Shrink the Constraint Violation Tolerance and Niche Radius**: $\varepsilon_t \rightarrow \varepsilon_{t+1}, \sigma_t \rightarrow \sigma_{t+1}$;
- Generate offspring population(set Q) from parent population(set P);
- Compute f and $G(\mathbf{x})$ values for each individual in Q;
- Update x_{best};
- $P = P \bigcup Q$;
- Compute $D(\mathbf{x})$ value for each individual in P;
- Apply the **nondominant sorting method** to truncate the population size of P from $2N_p$ to N_p;

end

Output $\mathbf{x}_{best}$.

E. Overall DMOA Algorithm

The pseudocode of DMOA is presented in Algorithm 1. In DMOA, each generation t contains all the stated parts.

1) A population in size N_p, $P_t = \mathbf{x}_{1,t}, \ldots, \mathbf{x}_{N_p,t}$.
2) Three objective values for each individual, $(F_T$, $G(\mathbf{x}_{i,t})$, $D(\mathbf{x}_{i,t}))$, $i = 1, \ldots, N_p$.
3) The niche radius σ_t.
4) The constraint violation tolerance e_t.
5) The best solution found so far x_{best}.

The density objective $D(\mathbf{x})$ and the shrinkage scheme are two important components in DMOA. Besides, DE is utilized as a search engine to generate offspring in each generation. Specific details of the proposed algorithm are given below.

1) *Initialization*: In the first generation, N_P individuals are randomly generated based on the minimum and maximum power outputs. Then, for each individual, its objective function value F_T and constraint violation $G_j(\mathbf{x})$ are calculated based on the given ED problem. Subsequently, the overall violation objective $G(\mathbf{x})$ in (4) is achieved, and the tolerance T_0 is initiated by the maximum value of $G(\mathbf{x})$.
2) *Shrink the Constraint Violation Tolerance and Niche Radius*: In each generation, the values of tolerance for overall constraint violation and niche radius are adjusted by (20). To shrink the niche radius, the parameters ε_t and T_0 are replaced by σ_t and σ_0, respectively.
3) *Nondominant Sorting Method*: When N_p offspring have been generated, the famous nondominated sorting method in NSGA-II [53] is used to truncate the population size from $2N_p$ to N_p. As described in Section IV-C, the dynamic Pareto dominance relation replaces the original Pareto dominance relation to compare two individuals in the diminishing environment.

VI. Experimental Study

In this section, the performance of DMOA is assessed on CEC 2006 test functions [54] and seven case studies of ED problems. The obtained results are compared with several state-of-the-art algorithms, respectively. Finally, the effectiveness of the density objective $D(\mathbf{x})$ and the shrinkage scheme are analyzed.

A. Parameter Setting

It is worth noting that DMOA does not introduce any new parameter. The population size N_p is set to 100. According to the recommendation in [21] and [55], the maximum generation number $t_{\max}$ for each CEC 2006 test function and test power system are set to 2400 and 3000, respectively. The parameters of the components of the proposed algorithm come from the original literatures where the corresponding operators are put forward. For each target vector in DE operators, F and Cr are randomly selected from the scaling factor pool $\{0.6, 0.8, 1.0\}$ and the crossover rate pool $\{0.1, 0.5, 1.0\}$, respectively. In the shrinkage scheme, cp is set to the recommended value $-3 - \log(T_0)/\log(0.5)$ in [52]. The initial niching radius σ_0 is set to $\sqrt{\sum_{j=1}^{N}(P_i^{\max} - P_i^{\min})}$ which is the longest Euclidean distance in the decision space. The tolerance parameter δ is set to 0.0001 [56] for the equality constraints. Each test function is run 25 independent trials.

B. Experiment With CEC 2006 Benchmark Functions

In this experiment, DMOA has been undertaken on 24 CEC 2006 test functions which have been widely used for algorithm evaluations. More details of these benchmarks can be found in [54]. Results for performance comparison with six state-of-the-art EAs are presented in Table S1 (see the supplementary material). The six EAs are APFGA [22], CCiALF [57], SAMODE [58], εDE [52], jDE-2 [59], and ECHT-EP2 [55]. The statistical results of the six algorithms are obtained from the original articles. The maximum fitness evaluations (Max_FEs) of APFGA, εDE, and jDE-2 are 5×10^5, while the Max_FEs of DMOA, ECHT-EP2, CCiALF, and SAMODE are 2.4×10^5. Because none of the algorithms can solve problems g20 and g22, comparisons and analyses are undertaken in the remainder 22 test functions.

It can be seen from Table S1 (see the supplementary material) that DMOA finds globally optimal solutions for all 22 test functions, while APFGA, CCiALF, SAMODE, εDE, jDE-2, and ECHT-EP2 can only find the globally optimal solutions for 14, 17, 14, 20, 14, and 14 test functions, respectively. It is worth noting that DMOA is not trapped in local optimum for all of the 22 test functions. On the two functions g02 and g17, the result of DMOA is substantially better than the other six algorithms. On the 15 functions g01, g03–g09, g11–g16, and g24, DMOA performs as good as the others. With regard to the mean value, the DMOA is better than APFGA, CCiALF, SAMODE, εDE, jDE-2, and ECHT-EP2 on eight, five, eight, two, eight, and eight test functions, respectively. Based on these comparisons, the DMOA outperforms the other six state-of-the-art algorithms.

TABLE I
Wilconxon Signed Ranks Test Results for DMOA Versus APFGA, CCiALF, SAMODE, εDE, jDE-2, and ECHT-EP2

Comparison	R^+	R^-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
DMOA vs APFGA	200.5	52.5	0.015576	Yes	Yes
DMOA vs CCiALF	163.0	68.0	0.095243	No	Yes
DMOA vs SAMODE	200.5	52.5	0.015576	Yes	Yes
DMOA vs εDE	148.0	105.0	0.475076	No	No
DMOA vs jDE-2	200.5	52.5	0.013948	Yes	Yes
DMOA vs ECHT-EP2	200.5	52.5	0.015159	Yes	Yes

Here, R^+ and R^- stand for the sum of ranks for the 22 test functions in which DMOA are better and worse to the against algorithm.

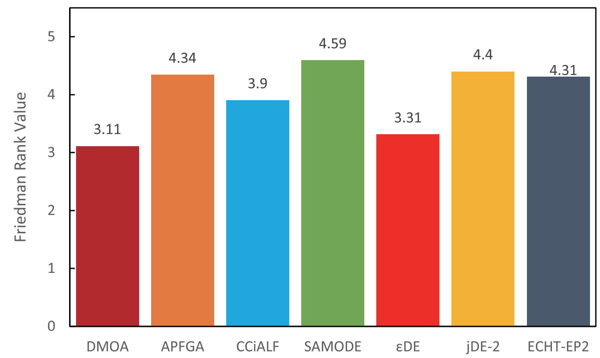

Fig. 4. Friedman ranks for DMOA, APFGA, CCiALF, SAMODE, εDE, jDE-2, and ECHT-EP2.

Furthermore, two nonparametric statistical tests, Friedman's and Wilcoxon's tests [60], are conducted via KEEL software [61] to figure out the significance of difference between the performance of DMOA and the others six EAs, using the mean of the cost values obtained in all 25 trials as the tested variable. As presented in Table I, pairwise comparisons between DMOA and the others six EAs are made through Wilcoxon's test. It is shown that DMOA offers a significant improvement over APFGA, CCiALF, SAMODE, jDE-2, and ECHT-EP2 at the significance level of $\alpha = 0.1$. Even though it fails to show significant difference between DMOA and εDE, DMOA can solve all the 22 test functions while εDE only solves 20. Moreover, the Max_FEs required by εDE are twice more than that required by DMOA. On the other hand, the result of Friedman's test presented in Fig. 4 also proves the superiority of DMOA over the other six EAs, for DMOA has the smallest Friedman rank value. Taking all these results into account, DMOA has a relatively better performance than the other six EAs.

C. Case Study

In this section, six real-world power systems with seven cases are adopted to assess the performance of the proposed DMOA. As for the first five cases, each of them includes an independent power system [13], [32]–[34], [62]. The last two cases come from the standard IEEE 30-bus test system [63], [64]. The practical features of the seven cases are summarized in Table II. Furthermore, to have a better performance assessment of DMOA, the minimum cost, average cost, standard deviation, and the best solution obtained in 25 independent trials are recorded for each case study. The

TABLE II
TEST POWER SYSTEM CONSIDERED FOR VALIDATION OF PROPOSED SOLUTION METHODOLOGY

Case	Number of Units	System Load Demand (WM)	Valve-point Loading	Ramp Rate	Prohibited-operating Zone	Multiple Fuel Options	Transmission-losses
1	6	1263	×	√	√	×	√
2	15	2630	×	√	√	×	√
3	10	2700	√	×	×	√	×
4	40	10500	√	×	×	×	×
5	140	49342	√	√	√	×	×
6	6	283.4	×	×	×	×	√
7	6	283.4	√	×	×	×	√

TABLE III
TEST VALUES OBTAINED BY GA, TSA, CBA, PSO, RDPSO, ST-IRDPSO, AND DMOA FOR SYSTEM 1

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Average time (s)	Standard Deviation
GA [32]	15459	15469	41.58	5.7E-02
TSA [65]	15451.63	15462.26	5.98	1.80+01
CBA [66]	15450.23	15454.76	0.704	2.96E+00
PSO [67]	15450.14	15465.83	6.82	1.01E+01
RDPSO [33]	15449.89	15458.01	0.707	1.36E+01
ST-IRDPSO [21]	15449.89	15450.70	0.727	1.41E+00
DMOA	**15444.74**	**15444.74**	35.55	5.78E-11

TABLE IV
TEST VALUES OBTAINED BY DE, GA, CPSO, PSO, SPSO, CTPSO, AND DMOA FOR SYSTEM 2

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Average time (s)	Standard Deviation
DE [33]	32718.8201	32966.4332	24.23	1.10E+02
GA [32]	32905.3592	33188.5443	62.73	8.89E+01
CPSO [68]	32835.2876	33021.8081	13.13	1.39E+02
PSO [67]	32735.6944	33039.0837	10.06	1.02E+02
SPSO [33]	32697.1431	32933.5688	16.39	1.37E+02
CTPSO [34]	32704.4514	32704.4514	22.50	0.00E+00
DMOA	**32692.34568**	**32692.34568**	57.34	6.21E-12

objective function value and the overall constraint violation against generation number for each case are also illustrated as the convergence analysis. Here the current best-known solution is considered to reflect the convergence speed of DMOA. In each generation, the current best-known solution is updated if there is a solution: 1) whose overall constraint violation is less than that of the best-known solution or 2) whose objective function value is less than that of the best-known solution and overall constraint violation is no more than that of the best-known solution.

1) Test Power System 1: This power system consists of six thermal units, 26 buses, and 46 transmission lines. Meanwhile, the prohibited operating zones, ramp rate limit, and transmission losses are involved in the constraints. The parameters, i.e., the cost coefficients, loss coefficients, and ramp rate limits data of this system, can be referred to [32]. Table III shows a comparison between the results obtained by DMOA and the other state-of-the-art algorithms (i.e., GA [32], TSA [65], CBA [66], PSO [67], RDPSO [33], and ST-IRDPSO [21]). All results obtained by the above algorithms satisfy the constraints of this test power system. The best results are highlighted in boldface. The convergence speed is plotted in Fig. S1 (see the supplementary material).

It can be observed from Table III that the minimum cost and average cost values obtained by DMOA are much better than those of the compared algorithms. The standard deviation indicates that DMOA is much more stable than the others in the 25 independent repeated trials. From Fig. S1 (see the supplementary material), it can be observed that the optimal solution can be located in less than 2500 generations, and both black and red cures decrease relatively stably. Actually, the shrinkage scheme has a great influence on the convergence speed because the individuals whose overall constraint violations are less than the violation tolerance can survive in the corresponding environment. Meanwhile, by utilizing multiobjective optimization techniques, the objective function and constraints can be minimized simultaneously. As a result, the search does not always converge to the solution with the minimum overall constraint violation, and hence its convergence speed is slightly impaired. Moreover, the details of the optimal solution are presented in Table S2 (see the supplementary material). According to each variable of the obtained optimal solution, this power system can satisfy all the requirements with the minimum generating cost.

2) Test Power System 2: This test power system contains a medium number of thermal units. The cost coefficients, loss coefficients, and ramp rate limits data are referred to [33]. There are 12 inequality constraints involved. Hence, this test power system is more difficult than system 1.

Table IV provides the minimum, average cost values, and the standard deviation achieved by DE [33], GA [32], CPSO [68], PSO [67], SPSO [33], CTPSO [34], and DMOA algorithms on this test power system. Table S3 (see the supplementary material) provides the details of the globally optimal solution obtained by DMOA. It is clear that DMOA obtains the minimum cost value in every independent trial. DMOA shows a significant improvement compared with the other six evolutionary algorithms, which further proves the stability of our proposed algorithm to solve ED problems. The CTPSO is also a stable evolutionary algorithm. However, based on the statistic results, it is easier than DMOA to get stuck in local optimal. Moreover, Fig. S2 (see the supplementary material) displays the convergence graph of our proposed algorithm.

3) Test Power System 3: In this case, the test power system with multifuels and valve-point effects are employed to evaluate the performance of our proposed algorithm. The power system consists of ten generating units and adopts three different fuel types. A comprehensive description of this power system can be referred to [13].

To have a better understanding, the proposed DMOA are compared with six state-of-the-art evolutionary algorithms, i.e., DE [69], IGA-MU [13], ARCGA [69], CCEDE [70],

TABLE V
TEST VALUES OBTAINED BY DE, IGA-MU, ARCGA, CCEDE, RDPSO, ST-IRDPSO, AND DMOA FOR SYSTEM 3

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Average time (s)	Standard Deviation
DE [69]	624.5146	624.5246	2.82	NA
IGA-MU [13]	624.5178	625.8692	7.32	NA
ARCGA [69]	623.8281	623.8431	NA	NA
CCEDE [70]	623.8288	623.8574	0.850	7.6E-03
CTPSO [33]	623.915	623.989	0.842	2.76E-02
RDPSO [33]	623.83	623.836	0.845	5.29E-03
DMOA	**623.8265**	**623.8326**	36.22	1.26E-02

TABLE VI
TEST VALUES OBTAINED BY CE-SQP, DE, CSA, IA-EDP, RDPSO, ST-IRDPSO, AND DMOA FOR SYSTEM 4

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Average time (s)	Standard Deviation
CE-SQP [71]	121412.88	121423.65	98.49	NA
DE [72]	121412.68	121439.89	NA	NA
CSA [73]	121412.54	121520.41	3.03	NA
IA-EDP [23]	121436.972	122492.701	1.09	1.82E+02
RDPSO [33]	121722.03	121972.90	3.28	2.43E+02
ST-IRDPSO [21]	**121412.535**	121443.792	3.54	3.34E+01
DMOA	121412.5443	**121420.8076**	66.42	9.01E+00

TABLE VII
TEST VALUES OBTAINED BY GWO, IDE, MTLA, CTPSO, RDPSO, ST-IRDPSO, AND DMOA FOR SYSTEM 5

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Average time (s)	Standard Deviation
GWO [74]	1559953.18	1560132.93	8.93	1.02E+00
IDE [75]	1564648.66	1564663.54	27.88	2.78E+01
MTLA [76]	1657951.9053	1657951.9053	0.04	NA
CTPSO [34]	1657962.73	1657964.06	100.00	7.31E+00
RDPSO [33]	1559708.679	1559775.46	5.98	1.05E+02
ST-IRDPSO [21]	1559708.679	1559751.215	6.13	5.69E+01
DMOA	**1559708.4550**	**1559720.9179**	111.06	2.82E+01

CTPSO [34], and RDPSO [33], and the comparison results are summarized in Table V. The best results are highlighted, and "NA" indicates that the data are not available. The details of the optimal solution are presented in Table S4 (see the supplementary material). It can be seen that DMOA obtains the best values for minimum and average costs in the 25 independent trials compared with the other six algorithms. As for the minimum cost value, DMOA significantly outperforms the cited heuristic algorithms, DE, IGA-MU, and RDPSO, and better than ARCGA, CCEDE, and ST-IRDPSO. This is very important for a multifuels power system, since it can economize remarkable consumption of the fossil fuel. As for the average cost value and standard deviation, DMOA is more stable than the other six algorithms to locate the globally optimal solution on test power system 3. The convergence graph of DMOA is plotted in Fig. S3 (see the supplementary material). It can be observed that the optimal solution can be located at a very fast speed.

4) Test Power System 4: This test power system is a medium-scale one. It has 40 generating units, and the valve-point effects are considered in all the units. The cost coefficients of this system can be referred to [62]. Since each cost function is nonconvex, there are a number of local optima in this test power system. Hence, this system is employed to investigate the global search ability of DMOA for multimodal ED problems. The six state-of-the-art evolutionary algorithms, CE-SQP [71], DE [72], CSA [73], IA-EDP [23], RDPSO [33], and ST-IRDPSO [21], are adopted for comparison. Table VI gives the statistical results achieved by these seven algorithms on this test power system.

It can be observed that the ST-IRDPSO finds the minimum cost value $\$121412.535h^{-1}$, followed by the DMOA whose minimum cost value is $\$121412.5443h^{-1}$, but when considering the average cost value, DMOA outperforms the other six EAs. The minimum cost values obtained by CE-SQP, CGRASP-SaDE, ST-IRDPSO, and DMOA have very slim differences. However, their standard deviation values are significantly different. DMOA achieves the best followed by ST-IRDPSO, indicating the higher stability of DMOA in solving medium-scale test power system with nonconvex cost functions. Table S5 (see the supplementary material) provides the power output of each generating unit of the best solution obtained by DMOA, and Fig. S4 (the supplementary material) displays the convergence graph.

5) Test Power System 5: To investigate our proposed algorithm in dealing with large-scale ED problems, a test power system with 140 generating units is utilized for testing. This system is derived from the power system of South Korea [34]. The valve-point effects, prohibited operating zones, ramp rate limits are all considered in this system. That is, this test power system contains not only large-scale generating units but also a large number of constraints. Hence, it is difficult for many heuristic algorithms to optimize. To evaluate the performance of our proposed algorithm, the six algorithms (i.e., GWO [74], IDE [75], MTLA [76], CTPSO [34], RDPSO [33], and ST-IRDPSO [21]) are adopted for comparison. Table VII gives the statistical results achieved by these seven algorithms on this test power system.

The best solution obtained by DMOA is presented in Table S6 (see the supplementary material) and the convergence graph is shown in Fig. S5 (see the supplementary material). It can be observed that the minimum cost value is $\$1559708.4550h^{-1}$, which is smaller than those of the other six evolutionary algorithms. The average cost value and the standard deviation indicate that the performance and stability of the DMOA are better than those of the six algorithms, especially better than the RDPSO, ST-IRDPSO, and GWO which have similar performance in minimum cost values. Furthermore, the average cost value obtained by DMOA is close to the minimum cost value, indicating that our proposed algorithm is more suitable for real-world power system for the reason that it is impossible to do a lot of independent repeated trials to find a minimum value by a given algorithm, since a large-scale power system optimization is usually time-consuming in real world.

Based on the excellent performance of DMOA on the five cases, it can be concluded that DMOA has the extraordinary ability to solve real-world ED problems with different characteristics.

6) Test Power System 6: In the following two cases, DMOA is tested on the standard IEEE 30-bus test system in which the constraints and objective function are more complex because

TABLE VIII
TEST VALUES OBTAINED BY ENHANCED GA, PSO, BBO, GRADIENT METHOD, EADDE, GSA, AND DMOA FOR SYSTEM 6

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Maximum Cost ($/h)	Average time (s)
GA [77]	802.06	NA	802.14	76
PSO [64]	800.41	NA	NA	NA
BBO [78]	799.111	799.198	799.204	11.02
Gradient Method [79]	804.853	NA	NA	NA
EADDE [80]	800.204	800.241	800.278	3.32
GSA [63]	798.675	798.913	799.028	10.785
DMOA	**793.836**	**795.788**	**798.700**	27.91

TABLE IX
TEST VALUES OBTAINED BY BBO, DE, PSO, MDE, GSA, AND DMOA FOR SYSTEM 7

Algorithms Applied	Minimum Cost ($/h)	Average Cost ($/h)	Maximum Cost ($/h)	Average time (s)
BBO [78]	647.743	647.764	647.792	11.94
DE [81]	650.8224	NA	NA	NA
PSO [64]	647.69	647.73	647.87	NA
MDE [82]	647.846	648.356	650.664	37.05
GSA [63]	646.848	646.896	646.938	10.27
DMOA	**643.599**	**644.538**	**646.468**	27.22

of some indirectly involved control variables. The system parameter settings are given in [63].

In this case, the objective function form is quadratic as formulated in (5b). The obtained minimum, average and maximum costs of DMOA is compared with enhanced GA [77], PSO [64], BBO [78], gradient method [79], EADDE [80], and GSA [63] are shown in Table VIII. The minimum fuel cost obtained by DMOA are \$793.836h^{-1}, and the corresponding solution is presented in Table S7 (see the supplementary material). The convergence graph of DMOA for the solution is shown in Fig. S6 (see the supplementary material). It can be seen that the overall constraint violation decreases to less than 1.0E-03 after 100 generations, which indicates that DMOA handles the constraints at a fast speed. For the comparison in Table VIII, the result of DMOA is 0.60% and 0.66% less than the previously reported minimum fuel cost \$798.675h^{-1} and \$799.111h^{-1} obtained by GSA and BBO, respectively. The average cost obtained by DMOA is \$795.788h^{-1} which is also better than the results of the compared algorithms.

7) Test Power System 7: In this case, objective functions of units 1 and 2 are dissevered as piecewise quadratic curves to simulate the valve-point effects and multiple fuels in (8). The corresponding cost coefficients for these two units are given in [63]. The remaining parameter settings and coefficients are set with the same values in case study 6.

Since the valve-point effects and multiple fuels are considered, there are several local optimal solutions, and hence, the search is susceptible to falling into local optima. The performance of the proposed DMOA is investigated in this case, and compared with BBO [78], DE [81], PSO [64], MDE [82], and GSA [63]. The comparison results are summarized in Table IX, and the control variables corresponding to the minimum fuel cost are given Table S8 (see the supplementary material). The minimum fuel cost and the average cost obtained by DMOA are \$643.599h^{-1} and \$644.538h^{-1}, respectively. The corresponding costs obtained by the followed approach GSA are \$646.848h^{-1} and \$646.896h^{-1}, respectively. The convergence graph of DMOA is plotted in Fig. S7 (see the supplementary material). It can be seen that the overall constraint violation curve descends consistently in the whole stage, while the fuel cost curve shows a faster rate of decline in the early stage than that in the middle and later stages. The two curves imply that DMOA not only focuses on handling constraints in the early stage but also optimizing the objective function, and therefore, chances are that the local optima can be avoided in the later stage.

D. Further Discussion

1) Effectiveness of the Density Objective: In our proposed algorithm, the fitness sharing is utilized to construct an additional objective that reflects the degree of crowding of each individual in its current generation. To investigate the validity of the density objective, a variant of DMOA, denoted as DMOA1, is generated. In DMOA1, there are only two objectives which are F_T and $G(\mathbf{x})$. The parameters in DMOA1 are kept unchanged except the niching radius which is also discarded.

Table S9 (see the supplementary material) presents the statistical results of the four test power systems for DMOA and DMOA1. It can be observed that both DMOA and DMOA1 can find the optimal solution with the minimum cost value, but the average cost value obtained by DMOA for each power test system is smaller than that obtained by DMOA1, especially for the test power systems 4 and 5. This indicates that the additional objective can improve the search ability to deal with the ED problems with the characteristic of multimodality, even for large-scale ones. In the evolutionary process, the number of nondominated solutions may increase because a dominated solution with regard to F_T and $G(\mathbf{x})$ can become a nondominated solution due to its lower value of $D(\mathbf{x})$. These nondominated solutions can survive in the next generation. By prolonging the survival of individuals with low values of $D(\mathbf{x})$, the population diversity can be enhanced. As a result, the local optima can be effectively avoided. Moreover, the density objective does not impair the search ability to locate global optimum for the other kinds of ED problems. Table S9 (see the supplementary material) shows that DMOA still outperforms DMOA1 on test power systems 1–3 with regard to the average cost values.

Furthermore, to investigate the convergence speed influenced by the density objective, convergence graphs for DMOA and DMOA1 on the test power systems 4 and 5 are plotted in Figs. 5 and 6. It can be seen that DMOA1 converges faster than DMOA with regard to objective function value and the overall constraint violation. This is because the density objective prolongs the survival of the sparse individuals, and thus the search takes more time to exploit the domains where these kinds of individuals stay. However, the density objective does not impair the convergence speed seriously. From Figs. 5 and 6, it can also be seen that the blue and orange curves just slightly lag behind the black and red curves, respectively. Furthermore, Tables III–IX show that DMOA is not the slowest with regard to the average time, except test power systems 3 and 5. Even though DMOA costs a little more time in test

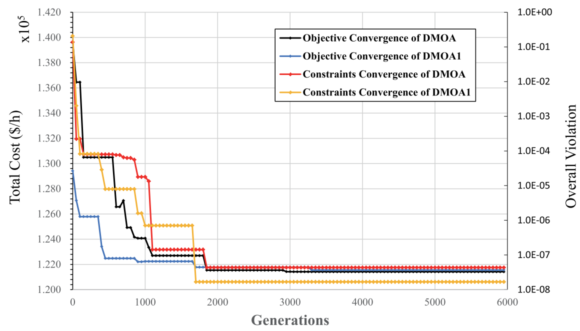

Fig. 5. Comparison of convergence performance between DMOA and DMOA1 for Test Power System 4.

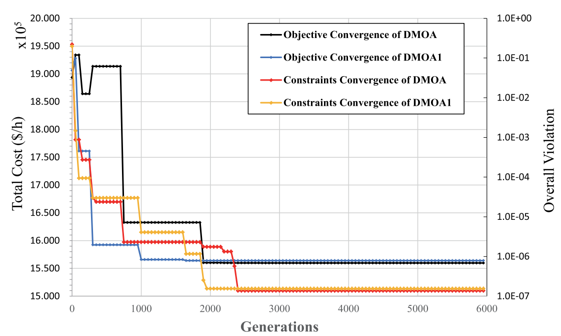

Fig. 6. Comparison of convergence performance between DMOA and DMOA1 for Test Power System 5.

power systems 3 and 5, the run time of DMOA is much less than the 5-min interval, which usually is the maximal run time requirement for ED problems [21]. It indicates that the density objective applied in DMOA is applicable to solve real-world ED problems.

2) Effectiveness of the Shrinkage Scheme: As mentioned in Section IV-B, the range of PF gradually becomes narrow under the control of the shrinkage scheme. To validate its effectiveness in DMOA, a variant named DMOA2 is generate to solve the five test power systems without the shrinkage scheme. Table S10 (see the supplementary material) provides the statistic results for DMOA and DMOA2.

As shown in Table S10 (see the supplementary material), the DMOA2 performs much worse than DMOA. DMOA2 cannot even consistently locate the feasible solutions in all independent trials. As plotted in Fig. 1, on the one hand, when the shrinkage scheme is not applied, the search should locate the endpoint A from the whole range of the initial PF. On the other hand, most points on the initial PF are infeasible solutions. Hence, it is difficult for DMOA2 to find the global optimum A accurately. This verifies that utilizing the multiobjective optimization techniques in the diminishing environment is very effective.

3) Effect of the Multiobjective Optimization in Shrinkage Scheme: It is interesting to observe in Figs. S1 and S2 (see the supplementary material), that the black curve does not always descend throughout the evolutionary process, while the red curve always declines during the whole stage. The reason is that before entering the feasible region, some individuals with small overall constraint violations have larger objective values. Nevertheless, DMOA does not fall into local optima with this kind of individuals. We attribute the above behavior to the fact that the multiobjective optimization takes its advantage of making a good balance between population diversity and convergence to amend the population generation by generation. Meanwhile, since the extended region enclosed by ε_t shrinks slightly in each generation, there is not a serious impairment caused by the environment change. Hence, multiobjective optimization has enough time to amend the search bias introduced by the attraction basins whose gradient values are very large. As a result, the search can avoid falling into local optima, and finally locate the global optimum.

VII. Conclusion

This article has proposed a dynamic multiobjective model to solve ED problems. To combine multiobjective optimization techniques with niching methods, the sharing function is utilized to construct an additional objective. The performance of DMOA is evaluated on 22 CEC 2006 test functions and seven real-world test power systems with different characteristics. The experimental results have demonstrated that DMOA performs better or at least competitive with the state-of-the-art algorithms, especially for the large-scale systems. Moreover, the effectiveness of the additional objective and the shrinkage scheme is also investigated on the five test power systems. The investigations validate that the shrinkage scheme is efficient in handling the constraints, and our proposed additional objective improves the search ability and the stability of DMOA for ED problems. Furthermore, the experiments also validate the feasibility of utilizing multiobjective optimization techniques to optimize an ED problem.

References

[1] V. Loia and A. Vaccaro, "Decentralized economic dispatch in smart grids by self-organizing dynamic agents," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 44, no. 4, pp. 397–408, Apr. 2014.

[2] B. Y. Qu, Y. S. Zhu, Y. C. Jiao, M. Y. Wu, P. N. Suganthan, and J. J. Liang, "A survey on multi-objective evolutionary algorithms for the solution of the environmental/economic dispatch problems," *Swarm Evol. Comput.*, vol. 38, pp. 1–11, Feb. 2018.

[3] X. He, D. W. C. Ho, T. Huang, J. Yu, H. Abu-Rub, and C. Li, "Second-order continuous-time algorithms for economic power dispatch in smart grids," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 9, pp. 1482–1492, Sep. 2018.

[4] G. Abbas, J. Gu, U. Farooq, M. U. Asad, and M. El-Hawary, "Solution of an economic dispatch problem through particle swarm optimization: A detailed survey-part I," *IEEE Access*, vol. 5, pp. 15105–15141, 2017.

[5] G. Abbas, J. Gu, U. Farooq, A. Raza, M. U. Asad, and M. E. El-Hawary, "Solution of an economic dispatch problem through particle swarm optimization: A detailed survey–part II," *IEEE Access*, vol. 5, pp. 24426–24445, 2017.

[6] W. Gao, A. Darvishan, M. Toghani, M. Mohammadi, O. Abedinia, and N. Ghadimi, "Different states of multi-block based forecast engine for price and load prediction," *Int. J. Elect. Power Energy Syst.*, vol. 104, pp. 423–435, Jan. 2019.

[7] A. A. El-Keib, H. Ma, and J. L. Hart, "Environmentally constrained economic dispatch using the Lagrangian relaxation method," *IEEE Trans. Power Syst.*, vol. 9, no. 4, pp. 1723–1729, Nov. 1994.

[8] J.-Y. Fan and L. Zhang, "Real-time economic dispatch with line flow and emission constraints using quadratic programming," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 320–325, May 1998.

[9] C.-L. Chen and N. Chen, "Direct search method for solving economic dispatch problem considering transmission capacity constraints," *IEEE Trans. Power Syst.*, vol. 16, no. 4, pp. 764–769, Nov. 2001.

[10] A. Farag, S. Al-Baiyat, and T. C. Cheng, "Economic load dispatch multiobjective optimization procedures using linear programming techniques," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 731–738, May 1995.

[11] O. Abedinia, M. Zareinejad, M. H. Doranehgard, G. Fathi, and N. Ghadimi, "Optimal offering and bidding strategies of renewable energy based large consumer using a novel hybrid robust-stochastic approach," *J. Clean. Prod.*, vol. 215, pp. 878–889, Apr. 2019.

[12] M. Saeedi, M. Moradi, M. Hosseini, A. Emamifar, and N. Ghadimi, "Robust optimization based optimal chiller loading under cooling demand uncertainty," *Appl. Therm. Eng.*, vol. 148, pp. 1081–1091, Feb. 2019.

[13] C.-L. Chiang, "Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1690–1699, Nov. 2005.

[14] J.-B. Park, K.-S. Lee, J.-R. Shin, and K. Y. Lee, "A particle swarm optimization for economic dispatch with nonsmooth cost functions," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 34–42, Feb. 2005.

[15] M. F. Zaman, S. M. Elsayed, T. Ray, and R. A. Sarker, "Evolutionary algorithms for dynamic economic dispatch problems," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1486–1495, Mar. 2016.

[16] N. Duvvuru and K. S. Swarup, "A hybrid interior point assisted differential evolution algorithm for economic dispatch," *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 541–549, May 2011.

[17] S. Hemamalini and S. P. Simon, "Dynamic economic dispatch using artificial bee colony algorithm for units with valve-point effect," *Int. Trans. Electr. Energy Syst.*, vol. 21, no. 1, pp. 70–81, Jan. 2011.

[18] S. Hemamalini and S. P. Simon, "Dynamic economic dispatch using artificial immune system for units with valve-point effect," *Int. J. Elect. Power Energy Syst.*, vol. 33, no. 4, pp. 868–874, 2011.

[19] E. Mezura-Montes and C. A. Coello Coello, "Constraint-handling in nature-inspired numerical optimization: Past, present and future," *Swarm Evol. Comput.*, vol. 1, no. 4, pp. 173–194, 2011.

[20] Y. Wang, B.-C. Wang, H.-X. Li, and G. G. Yen, "Incorporating objective function information into the feasibility rule for constrained evolutionary optimization," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 2938–2952, Dec. 2015.

[21] W. T. Elsayed, Y. G. Hegazy, M. S. El-bages, and F. M. Bendary, "Improved random drift particle swarm optimization with self-adaptive mechanism for solving the power economic dispatch problem," *IEEE Trans. Ind. Informat.*, vol. 13, no. 3, pp. 1017–1026, Jun. 2017.

[22] B. Tessema and G. G. Yen, "An adaptive penalty formulation for constrained evolutionary optimization," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 39, no. 3, pp. 565–578, May 2009.

[23] V. S. Aragón, S. C. Esquivel, and C. A. Coello Coello, "An immune algorithm with power redistribution for solving economic dispatch problems," *Inf. Sci.*, vol. 295, pp. 609–632, Feb. 2015.

[24] B.-C. Wang, H.-X. Li, J.-P. Li, and Y. Wang, "Composite differential evolution for constrained evolutionary optimization," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 7, pp. 1482–1495, Jul. 2019.

[25] B.-C. Wang, H.-X. Li, Q. Zhang, and Y. Wang, "Decomposition-based multiobjective optimization for constrained evolutionary optimization," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published.

[26] C. A. Coello Coello and E. M. Montes, "Constraint-handling in genetic algorithms through the use of dominance-based tournament selection," *Adv. Eng. Informat.*, vol. 16, no. 3, pp. 193–203, 2002.

[27] K. Deb, "An efficient constraint handling method for genetic algorithms," *Comput. Methods Appl. Mech. Eng.*, vol. 186, nos. 2–4, pp. 311–338, 2000.

[28] A. H. Aguirre, S. B. Rionda, C. A. Coello Coello, G. L. Lizárraga, and E. M. Montes, "Handling constraints using multiobjective optimization concepts," *Int. J. Numer. Methods Eng.*, vol. 59, no. 15, pp. 1989–2017, 2004.

[29] C. A. Coello Coello, "Treating constraints as objectives for single-objective evolutionary optimization," *Eng. Optim.*, vol. 32, no. 3, pp. 275–308, 2000.

[30] Y. Wang and Z. Cai, "Combining multiobjective optimization with differential evolution to solve constrained optimization problems," *IEEE Trans. Evol. Comput.*, vol. 16, no. 1, pp. 117–134, Feb. 2012.

[31] J.-Y. Ji, W.-J. Yu, Y.-J. Gong, and J. Zhang, "Multiobjective optimization with ε-constrained method for solving real-parameter constrained optimization problems," *Inf. Sci.*, vol. 467, pp. 15–34, Oct. 2018.

[32] Z.-L. Gaing, "Particle swarm optimization to solving the economic dispatch considering the generator constraints," *IEEE Trans. Power Syst.*, vol. 18, no. 3, pp. 1187–1195, Aug. 2003.

[33] J. Sun, V. Palade, X.-J. Wu, W. Fang, and Z. Wang, "Solving the power economic dispatch problem with generator constraints by random drift particle swarm optimization," *IEEE Trans. Ind. Informat.*, vol. 10, no. 1, pp. 222–232, Feb. 2014.

[34] J.-B. Park, Y.-W. Jeong, J.-R. Shin, and K. Y. Lee, "An improved particle swarm optimization for nonconvex economic dispatch problems," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 156–166, Feb. 2010.

[35] H. Singh and L. Srivastava, "Recurrent multi-objective differential evolution approach for reactive power management," *IET Gener. Transm. Distribution*, vol. 10, no. 1, pp. 192–204, 2016.

[36] B. Zhou, K. W. Chan, T. Yu, H. Wei, and J. Tang, "Strength Pareto multigroup search optimizer for multiobjective optimal reactive power dispatch," *IEEE Trans. Ind. Informat.*, vol. 10, no. 2, pp. 1012–1022, May 2014.

[37] F. Zaman, S. M. Elsayed, T. Ray, and R. A. Sarker, "An evolutionary framework for bi-objective dynamic economic and environmental dispatch problems," in *Intelligent and Evolutionary Systems*, Berlin, Germany: Springer, 2017, pp. 495–508.

[38] K. Miettinen, *Nonlinear Multiobjective Optimization*, New York, NY, USA: Springer, 1999.

[39] S. Dhanalakshmi, S. Kannan, K. Mahadevan, and S. Baskar, "Application of modified NSGA-II algorithm to combined economic and emission dispatch problem," *Int. J. Elect. Power Energy Syst.*, vol. 33, no. 4, pp. 992–1002, 2011.

[40] J. Cai, X. Ma, Q. Li, L. Li, and H. Peng, "A multi-objective chaotic particle swarm optimization for environmental/economic dispatch," *Energy Convers. Manag.*, vol. 50, no. 5, pp. 1318–1325, 2009.

[41] Y. Zhang, D.-W. Gong, and Z. Ding, "A bare-bones multi-objective particle swarm optimization algorithm for environmental/economic dispatch," *Inf. Sci.*, vol. 192, pp. 213–227, Jun. 2012.

[42] M. Basu, "Economic environmental dispatch using multi-objective differential evolution," *Appl. Soft Comput.*, vol. 11, no. 2, pp. 2845–2853, 2011.

[43] L. H. Wu, Y. N. Wang, X. F. Yuan, and S. W. Zhou, "Environmental/economic power dispatch problem using multi-objective differential evolution algorithm," *Elect. Power Syst. Res.*, vol. 80, no. 9, pp. 1171–1181, 2010.

[44] G. Adinolfi, G. Graditi, P. Siano, and A. Piccolo, "Multiobjective optimal design of photovoltaic synchronous boost converters assessing efficiency, reliability, and cost savings," *IEEE Trans. Ind. Informat.*, vol. 11, no. 5, pp. 1038–1048, Oct. 2015.

[45] I. Ciornei and E. Kyriakides, "A GA-API solution for the economic dispatch of generation in power system operation," *IEEE Trans. Power Syst.*, vol. 27, no. 1, pp. 233–242, Feb. 2012.

[46] J. G. Vlachogiannis and K. Y. Lee, "A comparative study on particle swarm optimization for optimal steady-state performance of power systems," *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp. 1718–1728, Nov. 2006.

[47] A. I. Selvakumar and K. Thanushkodi, "A new particle swarm optimization solution to nonconvex economic dispatch problems," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 42–51, Feb. 2007.

[48] R. Storn and K. Price, *Differential Evolution-A Simple and Efficient Adaptive Scheme for Global Optimization Over Continuous Spaces*, vol. 3. Berkeley, CA, USA: ICSI, 1995.

[49] S. Das, S. S. Mullick, and P. N. Suganthan, "Recent advances in differential evolution—An updated survey," *Swarm Evol. Comput.*, vol. 27, pp. 1–30, Apr. 2016.

[50] Q. Yang, W.-N. Chen, Y. Li, C. L. P. Chen, X.-M. Xu, and J. Zhang, "Multimodal estimation of distribution algorithms," *IEEE Trans. Cybern.*, vol. 47, no. 3, pp. 636–650, Mar. 2017.

[51] W.-J. Yu, J.-Y. Ji, Y.-J. Gong, Q. Yang, and J. Zhang, "A tri-objective differential evolution approach for multimodal optimization," *Inf. Sci.*, vol. 423, pp. 1–23, Jan. 2018.

[52] T. Takahama and S. Sakai, "Constrained optimization by the ε constrained differential evolution with gradient-based mutation and feasible elites," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, 2006, pp. 1–8.

[53] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

[54] J. J. Liang *et al.*, "Problem definitions and evaluation criteria for the CEC 2006 special session on constrained real-parameter optimization," *J. Appl. Mech.*, vol. 41, no. 8, pp. 8–31, 2006.

[55] R. Mallipeddi and P. N. Suganthan, "Ensemble of constraint handling techniques," *IEEE Trans. Evol. Comput.*, vol. 14, no. 4, pp. 561–579, Aug. 2010.

[56] S. Zeng, R. Jiao, C. Li, X. Li, and J. S. Alkasassbeh, "A general framework of dynamic constrained multiobjective evolutionary algorithms for constrained optimization," *IEEE Trans. Cybern.*, vol. 47, no. 9, pp. 2678–2688, Sep. 2017.

[57] B. Ghasemishabankareh, X. Li, and M. Ozlen, "Cooperative coevolutionary differential evolution with improved augmented Lagrangian to solve constrained optimisation problems," *Inf. Sci.*, vol. 369, pp. 441–456, Nov. 2016.

[58] S. M. Elsayed, R. A. Sarker, and D. L. Essam, "Multi-operator based evolutionary algorithms for solving constrained optimization problems," *Comput. Oper. Res.*, vol. 38, no. 12, pp. 1877–1896, 2011.

[59] V. L. Huang, A. K. Qin, and P. N. Suganthan, "Self-adaptive differential evolution algorithm for constrained real-parameter optimization," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, 2006, pp. 17–24.

[60] J. Derrac, S. García, D. Molina, and F. Herrera, "A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms," *Swarm Evol. Comput.*, vol. 1, no. 1, pp. 3–18, 2011.

[61] J. Alcalá-Fdez *et al.*, "Keel: A software tool to assess evolutionary algorithms for data mining problems," *Soft Comput.*, vol. 13, no. 3, pp. 307–318, 2009.

[62] N. Sinha, R. Chakrabarti, and P. K. Chattopadhyay, "Evolutionary programming techniques for economic load dispatch," *IEEE Trans. Evol. Comput.*, vol. 7, no. 1, pp. 83–94, Feb. 2003.

[63] S. Duman, U. Güvenç, Y. Sönmez, and N. Yörükeren, "Optimal power flow using gravitational search algorithm," *Energy Convers. Manag.*, vol. 59, pp. 86–95, Jul. 2012.

[64] M. A. Abido, "Optimal power flow using particle swarm optimization," *Int. J. Elect. Power Energy Syst.*, vol. 24, no. 7, pp. 563–571, 2002.

[65] S. Khamsawang and S. Jiriwibhakorn, "DSPSO–TSA for economic dispatch problem with nonsmooth and noncontinuous cost functions," *Energy Convers. Manag.*, vol. 51, no. 2, pp. 365–375, 2010.

[66] B. R. Adarsh, T. Raghunathan, T. Jayabarathi, and X.-S. Yang, "Economic dispatch using chaotic bat algorithm," *Energy*, vol. 96, pp. 666–675, Feb. 2016.

[67] S. Pothiya, I. Ngamroo, and W. Kongprawechnon, "Application of multiple tabu search algorithm to solve dynamic economic dispatch considering generator constraints," *Energy Convers. Manag.*, vol. 49, no. 4, pp. 506–516, 2008.

[68] J. Cai, X. Ma, L. Li, and P. Haipeng, "Chaotic particle swarm optimization for economic dispatch considering the generator constraints," *Energy Convers. Manag.*, vol. 48, no. 2, pp. 645–653, 2007.

[69] N. Amjady and H. Nasiri-Rad, "Solution of nonconvex and nonsmooth economic dispatch by a new adaptive real coded genetic algorithm," *Expert Syst. Appl.*, vol. 37, no. 7, pp. 5239–5245, 2010.

[70] M. Ghasemi, M. Taghizadeh, S. Ghavidel, and A. Abbasian, "Colonial competitive differential evolution: An experimental study for optimal economic load dispatch," *Appl. Soft Comput.*, vol. 40, pp. 342–363, Mar. 2016.

[71] J. Cai, Q. Li, L. Li, H. Peng, and Y. Yang, "A hybrid CPSO–SQP method for economic dispatch considering the valve-point effects," *Energy Convers. Manag.*, vol. 53, no. 1, pp. 175–181, 2012.

[72] W. T. Elsayed and E. F. El-Saadany, "A fully decentralized approach for solving the economic dispatch problem," *IEEE Trans. Power Syst.*, vol. 30, no. 4, pp. 2179–2189, Jul. 2015.

[73] D. N. Vo, P. Schegner, and W. Ongsakul, "Cuckoo search algorithm for non-convex economic dispatch," *IET Gener. Transm. Distribution*, vol. 7, no. 6, pp. 645–654, 2013.

[74] M. Pradhan, P. K. Roy, and T. Pal, "Grey wolf optimization applied to economic load dispatch problems," *Int. J. Elect. Power Energy Syst.*, vol. 83, pp. 325–334, Dec. 2016.

[75] M. Basu, "Improved differential evolution for economic dispatch," *Int. J. Elect. Power Energy Syst.*, vol. 63, pp. 855–861, Dec. 2014.

[76] T. Niknam, R. Azizipanah-Abarghooee, and J. Aghaei, "A new modified teaching-learning algorithm for reserve constrained dynamic economic dispatch," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 749–763, May 2013.

[77] A. G. Bakirtzis, P. N. Biskas, C. E. Zoumas, and V. Petridis, "Optimal power flow by enhanced genetic algorithm," *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 229–236, May 2002.

[78] A. Bhattacharya and P. K. Chattopadhyay, "Application of biogeography-based optimisation to solve different optimal power flow problems," *IET Gener. Transm. Distribution*, vol. 5, no. 1, pp. 70–80, 2011.

[79] K. Y. Lee, Y. M. Park, and J. L. Ortiz, "A united approach to optimal real and reactive power dispatch," *IEEE Trans. Power App. Syst.*, vol. PAS-104, no. 5, pp. 1147–1153, 1985.

[80] K. Vaisakh and L. R. Srinivas, "Evolving ant direction differential evolution for opf with non-smooth cost functions," *Eng. Appl. Artif. Intell.*, vol. 24, no. 3, pp. 426–436, 2011.

[81] A. A. A. El Ela, M. A. Abido, and S. R. Spea, "Optimal power flow using differential evolution algorithm," *Elect. Power Syst. Res.*, vol. 80, no. 7, pp. 878–885, 2010.

[82] S. Sayah and K. Zehar, "Modified differential evolution algorithm for optimal power flow with non-smooth cost functions," *Energy Convers. Manag.*, vol. 49, no. 11, pp. 3036–3042, 2008.

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Supplemental Materials for “Density-Enhanced Multiobjective Evolutionary Approach for Power Economic Dispatch Problem”

TABLE S1
EXPERIMENTAL RESULTS OBTAINED BY DMOA, APFGA, CCIALF, SAMODE, εDE, JDE-2 AND ECHT-EP2 FOR CEC 2006 TEST FUNCTIONS

Problem and its best known solution	FEs	DMOA	APFGA	CCiALF	SAMODE	εDE	jDE-2	ECHT-EP2
		2.4E+05	2.4E+05	2.4E+05	5.0E+05	5.0E+05	5.0E+05	2.4E+05
G01	Mean	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**
-15	std	0.00E+00	0.00E+00	2.40E-08	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G02	Mean	**-0.8036**	-0.8027	-0.793	-0.7987	**-0.8036**	-0.8015	-0.7998
-0.8036	std	1.1E-06	1.00E-04	8.30E-03	8.80E-03	1.75E-08	4.90E-03	6.20E-03
G03	Mean	**-1.0005**	**-1.0005**	**-1.0005**	**-1.0005**	**-1.0005**	-0.6624	**-1.0005**
-1.0005	std	8.31E-13	0.00E+00	1.70E-08	0.00E+00	2.96E-31	1.00E-01	0.00E+00
G04	Mean	**-30665.5386**	**-30665.5386**	**-30665.5386**	**-30665.5386**	**-30665.5386**	**-30665.5386**	**-30665.5386**
-30665.5387	std	3.67E-12	1.00E-04	9.80E-06	0.00E+00	0.00E+00	1.80E-12	0.00E+00
G05	Mean	**5126.4967**	5127.5423	**5126.4967**	**5126.4967**	**5126.4967**	5127.5726	**5126.4967**
5126.4967	std	2.22E-12	1.40E+00	9.10E-08	0.00E+00	0.00E+00	2.40E+00	0.00E+00
G06	Mean	**-6961.8138**	**-6961.8139**	**-6961.8139**	**-6961.8138**	**-6961.8139**	**-6961.8139**	**-6961.8139**
-6961.8139	std	0.00E+00	0.00E+00	5.10E-11	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G07	Mean	**24.3062**	**24.3062**	**24.3062**	24.3096	**24.3062**	**24.3062**	24.3063
24.3062	std	4.90E-08	0.00E+00	6.80E-07	1.50E-03	2.18E-15	1.40E-05	3.19E-05
G08	Mean	**-0.0958**	**-0.0958**	**-0.0958**	**-0.0958**	**-0.0958**	**-0.0958**	**-0.0958**
-0.0958	std	1.38E-17	0.00E+00	1.00E-15	0.00E+00	1.23E-32	3.80E-18	0.00E+00
G09	Mean	**680.6300**	**680.6300**	**680.6300**	**680.6300**	**680.6300**	**680.6300**	**680.6300**
680.63	std	1.05E-10	0.00E+00	5.40E-08	1.10E-05	0.00E+00	7.90E-14	2.60E-08
G10	Mean	**7049.2480**	7077.6821	**7049.2480**	7059.8134	**7049.2480**	**7049.2480**	7049.249
7049.248	std	2.10E-04	5.10E+01	6.00E-07	7.80E+00	4.24E-13	1.50E-06	6.60E-04
G11	Mean	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**
0.7499	std	1.11E-16	0.00E+00	2.00E-16	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G12	Mean	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**
-1.0	std	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G13	Mean	**0.0539**	**0.0539**	**0.0539**	**0.0539**	**0.0539**	0.7454	**0.0539**
0.0539	std	1.34E-14	0.00E+00	4.00E-06	1.70E-08	0.00E+00	2.20E-01	1.00E-12
G14	Mean	**-47.7648**	-47.7647	**-47.7648**	-47.6811	**-47.7648**	**-47.7648**	-47.7647
-47.7649	std	1.24E-10	1.00E-04	4.00E-08	4.00E-02	1.39E-15	6.40E-05	2.70E-05
G15	Mean	**961.7150**	**961.7150**	**961.7150**	**961.7150**	**961.7150**	961.7194	**961.7150**
961.715	std	5.68E-13	0.00E+00	1.80E-08	0.00E+00	0.00E+00	2.20E-02	2.00E-13
G16	Mean	**-1.9051**	**-1.9051**	**-1.9051**	**-1.9051**	**-1.9051**	**-1.9051**	**-1.9051**
-1.9051	std	9.32E-16	0.00E+00	9.70E-09	0.00E+00	1.58E-30	0.00E+00	1.12E-10
G17	Mean	**8853.5338**	8888.4876	8916.856	8853.5397	8853.5397	8888.5397	8853.5397
8853.5338	std	1.73E-10	2.90E+01	3.60E+01	1.10E-05	1.21E-27	3.60E-17	2.13E-08
G18	Mean	**-0.8660**	-0.8659	**-0.8660**	**-0.8660**	**-0.8660**	**-0.8660**	**-0.8660**
-0.866	std	1.01E-08	0.00E+00	3.50E-07	7.00E-07	2.18E-17	3.60E-17	1.00E-09
G19	Mean	**32.6555**	**32.6555**	32.6607	32.7573	32.6556	**32.6555**	32.6623
32.6556	std	1.40E-05	0.00E+00	2.30E-04	6.10E-02	1.26E-05	7.20E-10	3.40E-03
G21	Mean	**193.7245**	199.5158	193.7352	193.7713	**193.7245**	204.2026	193.7438
193.7245	std	7.54E-06	2.30E+00	1.20E-02	1.90E-02	3.34E-14	3.60E+01	1.60E-02
G23	Mean	**-400.0551**	-394.7627	-400.0536	-360.8176	**-400.0551**	-387.9531	-373.2178
-400.0551	std	1.64E-06	3.80E+00	5.00E-03	1.90E+01	1.11E-14	5.90E+01	3.30E+01
G24	Mean	**-5.5080**	**-5.5080**	**-5.5080**	**-5.5080**	**-5.508**	**-5.5080**	**-5.5080**
-5.508	std	8.88E-16	0.00E+00	1.00E-08	0.00E+00	2.52E-29	0.00E+00	1.80E-15

TABLE S2
OBTAINED OUTPUT POWER OF TEST SYSTEM 1 BY DMOA

P_1	P_2	P_3	P_4	P_5	P_6
447.1089	173.4528	264.1909	139.3316	165.8509	85.6368
Total Cost ($/h)			15444.7468		
Total power (MW)			1275.5722		
Total losses (MW)			12.5722		

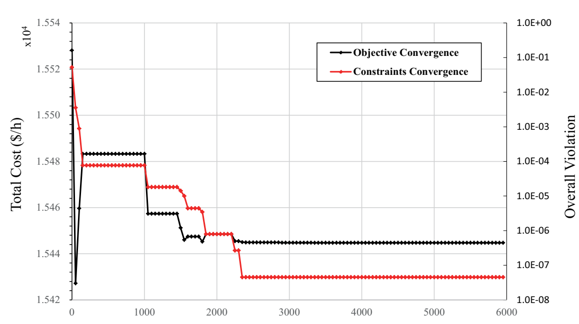

Fig. S1. Convergence graph of DMOA for Test Power System 1.

TABLE S3
OBTAINED OUTPUT POWER OF TEST SYSTEM 2 BY DMOA

Output Power										
$P_1 \sim P_{10}$	454.9999	379.9999	129.9999	130.0	169.9999	459.9999	429.9999	69.4563	60.1232	159.9999
$P_{11} \sim P_{15}$	79.9999	79.9999	25.0000	15.0000	15.00000	Total power (MW)	2659.5796	Total losses (MW)		29.5796
Total Cost ($/h)						32692.34568				

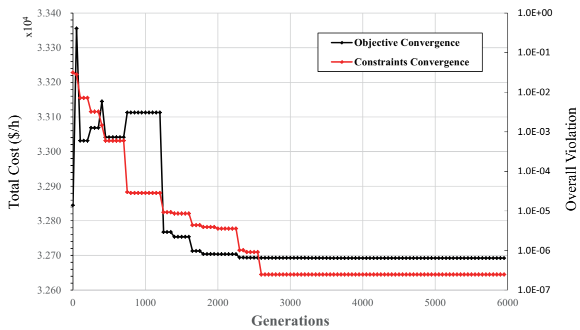

Fig. S2. Convergence graph of DMOA for Test Power System 2.

TABLE S4
OBTAINED OUTPUT POWER OF TEST SYSTEM 3 BY DMOA

Output Power										
$P_1 \sim P_{10}$	218.5939	211.4641	280.6570	239.6394	279.9345	239.6394	287.7274	239.6394	426.8357	275.8686
Total power (MW)			2700.0000			Total Cost ($/h)			623.8265	

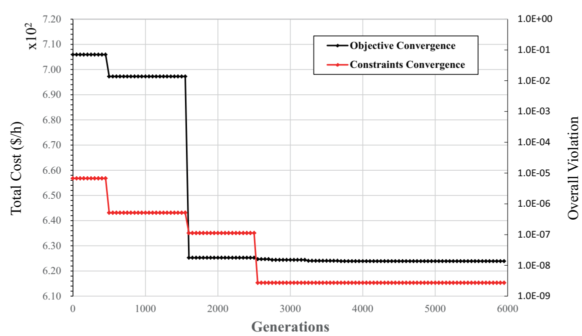

Fig. S3. Convergence graph of DMOA for Test Power System 3.

TABLE S5
OBTAINED OUTPUT POWER OF TEST SYSTEM 4 BY DMOA

Output Power										
$P_1 \sim P_{10}$	110.7998	110.7998	97.3999	179.7331	87.7999	139.9999	259.5996	284.5996	284.5996	130.0000
$P_{11} \sim P_{20}$	94.0000	94.0000	214.7597	394.2793	394.2793	394.2793	489.2793	489.2793	511.2793	511.2793
$P_{21} \sim P_{30}$	523.2793	523.2793	523.2793	523.2793	523.2793	523.2793	10.0000	10.0000	10.0000	87.7999
$P_{31} \sim P_{40}$	189.9999	189.9999	189.9999	164.7998	194.3976	199.9999	109.9999	109.9999	109.9999	511.2793
Total power (MW)			10500.0000			Total Cost ($/h)			121412.5443	

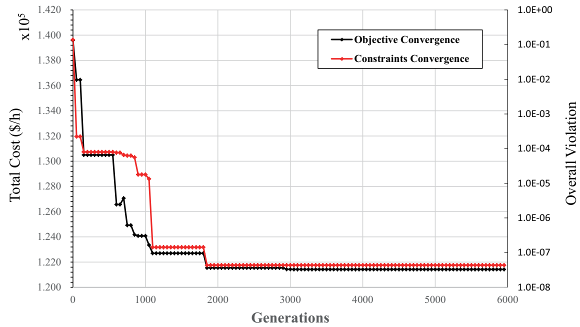

Fig. S4. Convergence graph of DMOA for Test Power System 4.

TABLE S6
OBTAINED OUTPUT POWER OF TEST SYSTEM 5 BY DMOA

Output Power										
$P_1 \sim P_{10}$	115.0454	188.9999	189.9998	189.9999	168.5398	189.9999	489.9999	489.9999	495.9999	495.9999
$P_{11} \sim P_{20}$	495.9999	495.9999	505.9999	508.9999	505.9999	504.9999	505.9999	505.9999	504.9999	504.9999
$P_{21} \sim P_{30}$	504.9999	504.9999	504.9999	504.9999	536.9999	536.9999	548.9999	548.9999	500.9999	500.9999
$P_{31} \sim P_{40}$	505.9999	505.9999	505.9999	505.9999	499.9999	499.9999	240.9999	240.9999	773.9999	768.9999
$P_{41} \sim P_{50}$	3.0000	3.0000	249.5848	246.9283	249.9999	249.9993	241.4809	249.9998	249.9991	249.9992
$P_{51} \sim P_{60}$	165.0005	165.0000	165.0001	165.0001	180.0000	180.0000	103.0001	198.0000	311.9999	281.3076
$P_{61} \sim P_{70}$	163.0000	95.0000	160.0004	160.0006	489.9987	196.0013	489.9988	489.9963	130.0000	234.7197
$P_{71} \sim P_{80}$	137.0002	325.4955	195.0003	175.0007	175.0000	175.0006	175.0001	330.0002	530.9999	530.9999
$P_{81} \sim P_{90}$	396.8984	56.0000	115.0000	115.0000	115.0000	207.0000	207.0000	175.0003	175.0002	175.0003
$P_{91} \sim P_{100}$	175.0000	579.9999	644.9999	983.9999	977.9999	681.9999	719.9999	717.9999	719.9999	963.9999
$P_{101} \sim P_{110}$	957.9999	1006.9999	1005.9999	1012.9999	1019.9999	953.9999	951.9999	1005.9999	1012.9999	1020.9999
$P_{111} \sim P_{120}$	1014.9999	94.0000	94.0000	94.0000	244.0000	244.0001	244.0001	95.0001	95.0001	116.0000
$P_{121} \sim P_{130}$	175.0000	2.0000	4.0000	15.0000	9.0000	12.0000	10.0000	112.0000	4.0000	5.0000
$P_{131} \sim P_{140}$	5.0000	50.0000	5.0000	42.0000	42.0000	41.0000	17.0000	7.0000	7.0000	26.0002
Total power (MW)			49342.0000			Total Cost ($/h)			1559708.4550	

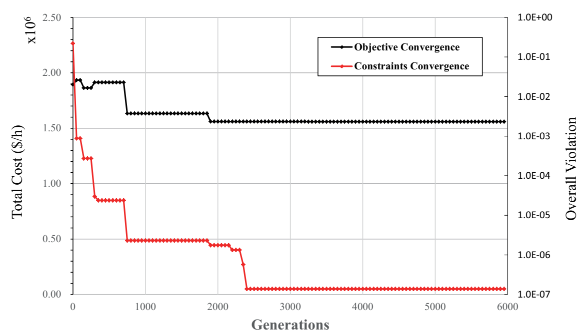

Fig. S5. Convergence graph of DMOA for Test Power System 5.

TABLE S7
THE OBTAINED BEST RESULT OF TEST SYSTEM 6 BY DMOA

P_1	P_2	P_5	P_8	P_{11}	P_{13}	V_1	V_2	V_5	V_8	V_{11}	V_{13}	t_{11}	t_{12}	t_{15}	t_{36}
175.86	48.09	20.48	21.40	11.16	12.04	1.06	1.043	1.01	1.01	1.08	1.07	0.978	0.969	0.932	0.968
Total Cost ($/h)			793.836		Total power (MW)			289.066		Total losses (MW)			5.666		

TABLE S8
THE OBTAINED BEST RESULT OF TEST SYSTEM 7 BY DMOA

P_1	P_2	P_5	P_8	P_{11}	P_{13}	V_1	V_2	V_5	V_8	V_{11}	V_{13}	t_{11}	t_{12}	t_{15}	t_{36}
139.65	54.90	23.82	33.41	18.07	17.50	1.06	1.043	1.01	1.01	1.08	1.07	0.978	0.969	0.932	0.968
Total Cost ($/h)			643.599		Total power (MW)			287.371		Total losses (MW)			3.971		

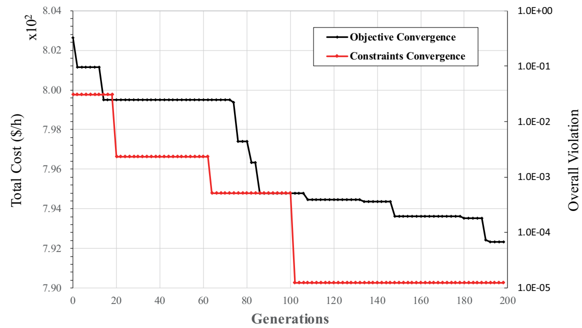

Fig. S6. Convergence graph of DMOA for Test Power System 6.

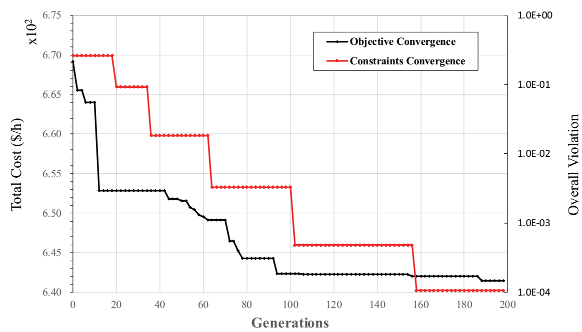

Fig. S7. Convergence graph of DMOA for Test Power System 7.

TABLE S9
TEST VALUES OBTAINED BY DMOA AND DMOA1 FOR FIVE TEST POWER SYSTEMS

Algorithm	DMOA			DMOA1		
Case	Minimum Cost ($/h)	Average Cost ($/h)	Standard Deviation	Minimum Cost ($/h)	Average Cost ($/h)	Standard Deviation
1	**15444.7455**	**15444.7455**	5.78E-11	**15444.7455**	15444.9547	4.79E-01
2	**32692.3456**	**32692.3456**	6.21E-12	**32692.3456**	**32692.3456**	4.42E-12
3	**623.8265**	**623.8326**	1.26E-02	**623.8265**	623.8348	3.82E-02
4	**121412.5443**	**121420.8076**	9.01E+00	**121412.5443**	121522.1495	7.75E+01
5	**1559708.4550**	**1559720.9179**	2.82E+01	**1559708.4550**	1559944.3592	6.91E+02

TABLE S10
TEST VALUES OBTAINED BY DMOA AND DMOA2 FOR FIVE TEST POWER SYSTEMS

Algorithm	DMOA			DMOA2		
Case	Minimum Cost ($/h)	Average Cost ($/h)	Standard Deviation	Minimum Cost ($/h)	Average Cost ($/h)	Standard Deviation
1	**15444.7455**	**15444.7455**	5.78E-11	15462.0410	15511.7471*	3.55E+01
2	**32692.3456**	**32692.3456**	6.21E-12	32889.6254	33121.9125*	1.16E+02
3	**623.8265**	**623.8326**	1.26E-02	674.0701	750.1978*	5.32E+01
4	**121412.5443**	**121420.8076**	9.01E+00	128065.4524	138713.8336*	6.41E+03
5	**1559708.4550**	**1559720.9179**	2.82E+01	1733421.0779	1820298.8586*	4.49E+04

Results with * means that there are infeasible solutions over 25 independent runs.